TELEGRAPH
TRANSMISSION THEORY

TELEGRAPH TRANSMISSION THEORY

BY

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WITH A FOREWORD BY

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FOREWORD

I⊤ is noteworthy that telegraphy, which, at the turn of the century, with the advent of the telephone, appeared to have reached its zenith, was probably never the source of more general interest than it is to-day. During the war years the range of the utilization of telegraphy transcended any previous ideas as to the possibility of its usefulness.

Although the widespread use of the teleprinter as a means of the rapid interchange of business communications accounts in large measure for this interest, the development of alternating current methods of telegraph transmission has contributed not a little by the means it has afforded for the economical provision of high quality long-distance telegraph channels.

In co-ordinating the technical advance of telegraphy the International Consultative Telegraph Committee (C.C.I.T.) has played a notable part, and its agreed definitions and directions have facilitated telegraph communication not only in this country but throughout the European comity of nations.

The older theories of telegraph transmission were naturally limited to ideas associated with direct currents, since they belonged to an era prior to that of the telephone, and the subsequent development of alternating current transmission theory. A treatment of telegraph transmission incorporating modern ideas and linking the old and new techniques, written for the student, is undoubtedly overdue.

This book, which is in reality the advanced issue of a part of a much larger work, lays down the basis of modern telegraphy in a way suitable for students; and, as one who is deeply interested in the progress of telegraphy in all its aspects, I commend it to the attention of all who wish to build their knowledge of telegraphy on firm foundations.

<div align="right">F. E. NANCARROW</div>

PREFACE

THE subject of this short work is one on which very little has ever been published in a form suitable and accessible to the less advanced students of Telegraphy. Practical descriptions of telegraph apparatus and equipment are on the whole well covered, but in most published information relating to theory the treatment is generally too advanced for the student whose main interest lies in the practical application of theory. Telegraph transmission theory is almost wholly concerned with transient conditions, the mathematical treatment of which is notoriously more complex than that of steady-state conditions which, fortunately, largely suffice for telephone transmission problems.

In common with other branches of electric circuit theory some knowledge of mathematics is necessary to an understanding of telegraph transmission, but it is possible to acquire a good working knowledge without recourse to the advanced mathematics required for the strict solution of telegraph transmission problems, and the theory has been developed accordingly in the present treatment.

The advent of alternating current telegraphy has brought into the field of telegraph transmission the theory of a.c. transmission, including filter theory, thus introducing the steady-state solutions already mentioned. This widens the scope considerably, a fact which is not at all without benefit, because transient conditions can be approached and explained through the solutions for the steady-state, and a much more satisfactory treatment, from the point of view of the less advanced student, becomes possible.

While the need for some knowledge of mathematics cannot be avoided if transmission theory is to be understood, the aim has been to reduce this need to a minimum, and graphical examples relating to simple artificial lines have been used to a great extent. The proofs of all formulae have been excluded from the main text and are given at the end. This allows better continuity of the practical treatment, on which emphasis has been laid, but must not be construed as implying that a knowledge and understanding of the proofs is not of great value. A full appreciation of the meaning and application of theory is

hardly possible without a knowledge of the basis on which the theory is founded.

The treatment of a.c. transmission theory is not so comprehensive as that required for the student of telephone transmission and has been restricted to those aspects directly relevant to telegraphy.

Some notes have been included dealing with recent developments in multi-channel voice-frequency telegraphy mainly arising from the application of this system of working to radio links. The cases dealt with, namely, multi-tone working and frequency modulation, are as yet in an early stage of development and it is too soon to say what will be the outcome in this new field of application. Frequency modulation, in particular, which in concept seems very attractive, loses some of its charm on closer examination, and amplitude modulation is likely to maintain undisputed supremacy for many of the uses of multi-channel telegraphy.

The matter was originally prepared for inclusion in a comprehensive textbook on Telegraphy, and therefore assumes knowledge of, or access to, practical descriptions of such common items of modern telegraphy as the teleprinter and multi-channel voice-frequency telegraph equipment.

It is hoped, however, that the publication of this booklet in advance of the complete textbook will do something towards filling a long felt gap in telegraph literature, and that the method of treatment adopted will help towards a fuller understanding of an otherwise rather intractable subject for the general student.

PUBLISHERS' SPECIAL NOTE

THIS treatise on the theory of Telegraph Transmission is intended to form part of a new and completely revised edition of Herbert's *Telegraphy* which is now in course of preparation. The striking and radical changes which, of recent years, have completely transformed the whole art of telegraphy, have necessarily involved considerable changes in the book, which for so many years has stood as a standard compendium of telegraph practice. This and the intervention of the War, has unavoidably delayed the publication of the new edition; but it has been thought that the advance release of the Section now issued, covering as it does an aspect of the subject which is, perhaps, least well documented, will meet an immediate need until such time as the complete edition becomes available.

CONTENTS

CONTENTS

TELEGRAPH TRANSMISSION THEORY

SECTION 1

TELEGRAPH TRANSMISSION

BEFORE studying the phenomena associated with the electrical transmission of telegraph signals, it is desirable that the fundamental basis of telegraph signalling should be properly appreciated.

Ordinary telegraph systems are designed to permit the electrical communication of intelligence with the minimum possible requirements in regard to the electrical characteristics of the transmission link. This is achieved by making use of a simple arbitrary signal code to represent letters, figures, and signs, which avoids the need for transmitting the complex wave-forms required for telephony or facsimile reproduction. By this means, at the expense of some complication of the transmitting and receiving apparatus, compared with that required for telephony, substantial economies in line plant are secured.

The telegraph signals are formed at the sending end of the telegraph channel by making simple changes in the electrical conditions such as, for example, switching on and off, or reversing, a steady potential, or switching on and off a voice-frequency tone, the important fact being that the significance of the signals so formed is fixed by the time intervals between successive switchings or reversals. At the receiving end the transmission is completed by the recognition of these time intervals by the receiver, thereby enabling the required characters to be recorded.

It is important that the "time-interval" characteristic of telegraph signals should be appreciated, because this feature is the basis of telegraph transmission technique and the various factors of signalling speed, distortion, and reception margin are all defined in relation to the basic time intervals on which the signals are built up.

1

Definitions of Terms Relating to Telegraph Transmission. In order to provide a practical basis for the determination of the transmission qualities of telegraph channels and apparatus the C.C.I.T. (Comité Consultatif International Télégraphique) has produced a number of definitions relating to telegraph transmission. In order to assist in framing the definitions a number of terms, the definitions of which follow, have been introduced, and the use of these terms is of considerable assistance in discussing both practical and theoretical transmission problems.

(*Telegraph modulations* are the operations performed by the transmitter in producing a telegraph signal.)

(*Characteristic instants of modulation* are the instants at which the characteristic changes occur.)

(*Elements of modulation* are the intervals between the characteristic instants. The elements of modulation are frequently called *signal elements.*)

DOUBLE-CURRENT TELEGRAPH SIGNAL

MARKING

SPACING

a   b   c   d   e   f

TIME

a,b,c,d, etc are "characteristic instants of modulation".
Time intervals a-b, b-c, c-d, etc are "elements of modulation".

Fig. 1. Definition of Telegraph Signals

The foregoing terms are illustrated diagrammatically in Fig. 1.

The essential characteristic of the transmitted electrical wave lies in the changes in character produced by the characteristic modulations, and although the changes may be effectively instantaneous at the sending end, they lose their sudden character during their propagation through the line and terminal equipments. This delay in the building up of the received signal to its final state is mainly due to the transient effects which, in general, accompany the changes in the transmitted wave. On account of the distortion of the transmitted wave, the recognition, at the receiving end, of the time intervals between the original changes or modulations may be a matter of some difficulty. In order to complete the transmission, therefore, it is usual to restore the abrupt nature of the changes by some form of relay. The latter should have the property of producing locally, instantaneous electrical or mechanical changes at instants corresponding to those at which the amplitude of the operating current is equal to some predetermined value.

The foregoing effects are illustrated in Fig. 2, which refers to the transmission of double-current telegraph signals on a

simple telegraph circuit terminating on a polarized relay. The first graph shows the modulations produced by the transmitter, which consist of voltage reversals. The short periods
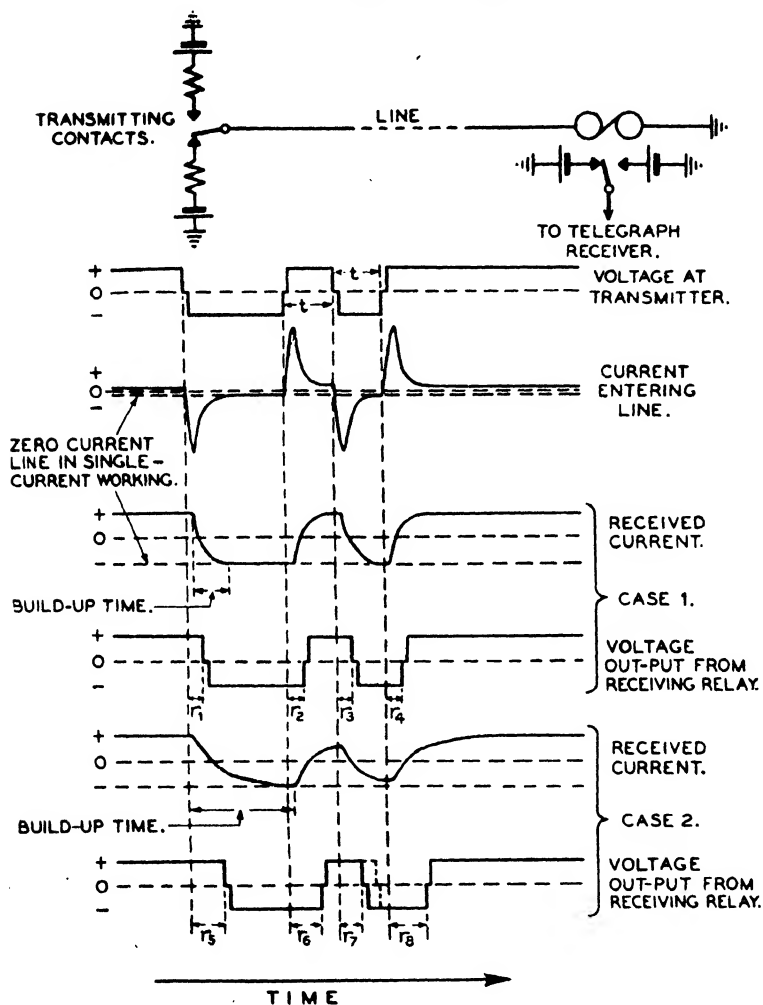
FIG. 2. GRAPHS OF SENT AND RECEIVED SIGNALS
SHOWING BUILD-UP TIMES AND RESTITUTION DELAYS

of zero voltage, shown in the voltage graphs, occur when the tongue is moving from one fixed contact to the other. The second graph indicates the current flowing into the line. It will be seen that the peak value of the surge current is far

in excess of the steady current, the current scale being one-fifth of that used for the received current. Two cases of received current have been given. In case 1 the duration of the transient conditions—that is, the build-up time—is slightly less than the duration of the shortest signal element, whereas in case 2, which gives the received current for a line of greater length, the build-up time is approximately twice the duration of the shortest element. If the duration of the transient or build-up time is shorter than that of the shortest element of modulation, the shape of the received signals will be unaffected by previous signals under all conditions, as the received current will always reach the steady state before the commencement of the next element. Assuming this to be so, then in the absence of external disturbing influences the response of the system to a modulation will always be the same, and the instantaneous changes produced by a perfect receiving relay will always occur with a constant delay after the corresponding operation at the origin of transmission. Under these circumstances the received signals will be in exact accordance with the operation of the transmitter, and the system will be telegraphically distortionless. For example, in case 1 of Fig. 2, the received current always reaches the steady condition within the length of the signal elements, and therefore no distortion occurs, but in case 2 this is not so and as a result the second signal element is shortened and the third element lengthened. The way in which this distortion is measured and expressed is dealt with later.*

Telegraph Speed : The Baud. The degree of imperfection of the received signals is to a large extent dependent upon the relation between the duration of the transient effects and the duration of the shortest element of modulation. These two factors are therefore of primary importance in telegraph transmission, since the maximum rate at which information

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* Actually, distortionless transmission is still obtainable when the duration of the transients exceeds the shortest modulation element, it only being necessary that transients due to one modulation shall have died away before the receiving relay operates in response to the next following modulation. This requirement sets a theoretical maximum transient time equal to twice the duration of the shortest modulation element. To secure this limiting condition in double-current working, the requirements are that the receiving relay shall always change over when the current passes through zero, and the time taken for the current to pass from steady positive to zero is equal to the time taken from zero to steady negative. In practice such perfection is neither obtainable nor essential, but in all systems there is some limit to the permissible departure of the received signals from perfect agreement with those at the origin.

may be passed over a channel is inversely proportional to the duration of the shortest element of modulation.) These facts have led to the use of the shortest element of modulation as the basis for the numerical expression of telegraph speed, which is defined as the inverse of the duration (expressed in seconds) of the shortest element of modulation. The unit of telegraph speed therefore corresponds to one minimum element per second and is called the *baud*. In Fig. 2 the duration of the shortest element of modulation is shown as t sec, and the speed in bauds is therefore $1/t$.

The following examples show the application of this definition to some well-known telegraph systems.

Teleprinter. $7\frac{1}{2}$-unit code, operating at 400 letters per min, (i.e. speed of transmitting shaft is 400 r.p.m.).

Duration of one letter $= 60/400 = 0\cdot15$ sec.

,, ,, element $= 0\cdot15/7\cdot5 = 0\cdot02$ sec.

Telegraph speed $= 1/0\cdot02 = 50$ bauds.

(*Note.* Assuming an average word of 5 letters and 1 space signal, 400 letters per min $= 66\cdot6$ words per min.)

Wheatstone automatic. Assume that the transmitter is operating at a speed of 20 ft of perforated tape per min. As there are 10 centre holes per in., this is equivalent to 40 centre holes per sec, or 40 dots per sec. As each dot is followed by a spacing period equal to the duration of the dot, the length of a dot signal, which is the shortest element in the code, is $\frac{1}{80}$ sec, and the telegraph speed is therefore 80 bauds. To convert this to words per minute, because Morse is an unequal length code, it is necessary to know the average number of signal elements in a word of five letters with one word space. For Continental Morse and plain English this is $51\cdot6$, but for telegraph traffic it is usually taken as 48. Thus a telegraph speed of 80 bauds is equivalent to $(80/48) \times 60 = 100$ words per min.

Baudot Multiplex. Quadruple system with two correcting segments, distributor running at 180 r.p.m. Number of segments on transmitting ring $= 4 \times 5 + 2 = 22$.

Duration of shortest element $= (60/180) \times (1/22) = 1/66$ sec.

Telegraph speed $= 66$ bauds.

Cable Code. Automatic transmitter operating at 1 200 centre holes per min. In this case there is one element (dot, dash, or space) per centre hole so that the telegraph speed is 20 bauds. For cable traffic the average number of elements per character is taken as $3\cdot7$, or $22\cdot2$ per word of five letters and

one space. In this case therefore 20 bauds is equivalent to (20/22·2) × 60 = 54 words per min.

In cable transmission, to improve the signal shape it is often arranged for the battery only to be applied to the line for from 75 to 85 per cent of the duration of the signal elements. This does not however affect the baud speed as the number of elements per second is still equal to the number of centre holes per second.

Telegraph Distortion. The process of reproducing the characteristic instants of modulation at the receiving end by means of some form of relay has been given the name *restitution of modulation*, while the time interval between an instant of modulation and the corresponding instant of restitution is called the *restitution delay*. In the cases illustrated in Fig. 2, graphs have been drawn giving the output from the receiving relay tongue, assuming that the tongue of the relay always changes over when the current has reached a value equal to half the steady current. The time delay of the resulting signals with respect to the original signals is indicated by r_1, r_2, r_3, r_4 in the first case, and r_5, r_6, r_7, r_8 in the second case. These time intervals are the restitution delays. In case 1 these delays are all equal, for the reasons already explained; the timing of the received signals is therefore in exact accordance with that of the sent signals, and the system is telegraphically distortionless. In case 2, r_7 is considerably shorter than r_5, r_6 and r_8, and distortion results to the corresponding signal elements. (In practice, the restitution delays may vary appreciably from one instant to another, and with excessive deviation from constancy the correct interpretation of the received signals is either impossible or involves unduly meticulous adjustments.) The accuracy and stability of operation of a telegraph circuit is dependent on the extent of these deviations, and their value is therefore used to express the quality of transmission. The method adopted will be described by reference to Fig. 3.

The sent signal has been superimposed, in an arbitary position on the received signal, to show up more clearly the relative displacements of the instants of modulation, the displacements being indicated by the shaded portions. (The distortion is always expressed as a fraction or percentage of the duration of the shortest modulation element,) indicated in the figure by t. Thus the distortion of the first element, that is, the total relative displacement of the two corresponding modulation instants, is given by $r_1 - r_2 = d_1 + d_2$ or $[(r_1 - r_2)/t] \times 100$ per cent. The value of distortion usually

quoted is that given by the maximum relative displacement of any two instants of modulation in the signals concerned. In the case illustrated this is therefore $[(r_1 - r_5)/t] \times 100$ per cent, because r_1 is the greatest and r_5 the smallest restitution delay. In practice, when measuring the distortion on a telegraph channel it is usual to use a train of signals composed of those which will suffer the greatest distortion, so that the result will give the maximum distortion likely to be experienced.

Telegraph distortion is therefore defined as follows—

The distortion of received telegraph signals is given by the

DISTORTION OF FIRST ELEMENT $= r_1 - r_2 = d_1 + d_2$
MAXIMUM DISTORTION $= r_1 - r_5 = d_1 + d_5$
PERCENTAGE DISTORTION $= \frac{r_1 - r_5}{t} \times 100$
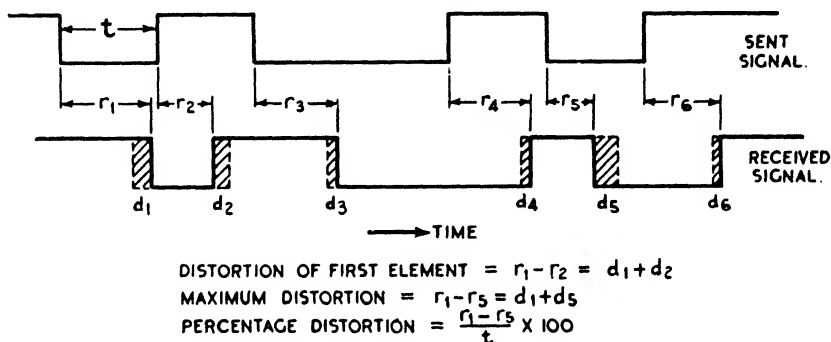
FIG. 3. DISTORTED TELEGRAPH SIGNAL ILLUSTRATING DEFINITION OF DISTORTION

maximum difference in the restitution delays of the characteristic instants of modulation, and is expressed numerically as the ratio of this difference to the duration of the shortest modulation element. (In practice this ratio is usually written as a percentage.)

It should be noted that this definition relates to a telegraphic link comprising a transmitting device and a receiving relay, and bears no direct relation to the actual shape of the signal currents in the line. In practice the link would also include any signal shaping networks or devices normally used, as without these the values would have no practical significance.

It has been found convenient to classify telegraph distortion as follows—

(a) *Characteristic Distortion.* The distortion occurring consistently with any given series or combination of signal elements.

(b) *Fortuitous Distortion.* The distortion due to irregularities in any part of the circuit or apparatus, or interference from external sources.

(c) *Bias Distortion.* This is due to either the marking or

spacing elements of the signals being consistently lengthened due to asymmetry in the transmitting or receiving apparatus.

Transmitted telegraph signals may be imperfect at their origin due to transmitter distortion. This can be measured and given a numerical value as a ratio, in the same way as is done for telegraph distortion generally, by comparing the actual instants of modulation with those which would occur with perfect signals. Teleprinter transmitters are usually adjusted so that the distortion does not exceed 5 per cent.

Margin of Receiving Apparatus. All telegraph receivers are adapted to permit a certain amount of distortion in the received signals. This is necessary so as to allow for transmitter and channel distortion and possible slight difference of speed between the sending and receiving machines. The degree of distortion that a receiver will permit is termed its *margin*, and is defined as follows—

The margin of a telegraph receiver is measured as the maximum distortion which may occur on any or all the signals which may be applied to it, compatible with perfect registration of all the symbols for which it is designed.

In the Creed Teleprinter the necessary margin is secured by the fact that the finger setting blade, which oscillates in response to the incoming signals, only needs to remain in contact with the finger setting pin for a very short time (about 2 msec) in order to set a selecting finger. There is therefore only a short period during each signal element when the electro-magnet armature and finger setting blade must be in the correct position to ensure the setting up of the proper selection, and considerable variation in the timing of the signal elements can occur before errors result.

SECTION 2

LINE TRANSMISSION THEORY : DIRECT CURRENT

IT will have been realized from the previous section that the function of a direct current telegraph line is to transmit the impulses resulting from the sudden changes in voltage at the sending end. If the transient time, that is, the time required for the current to build up to the steady value at the receiving end, is less than the duration of the shortest signal element, there will be no telegraph distortion due to the line, and the problem is merely one of providing sufficient current to secure reliable operation of the receiving relay. If, however, the build-up time is longer than this there may be some distortion. The stage at which distortion arises is mainly dependent upon the ratio of the operating current of the relay to the steady current, but it would usually be reached when the build-up time approaches 1·5 times the duration of the shortest signal element. Thus in those cases where the line conditions become of importance in determining the quality of transmission, the signals are almost wholly composed of *transients*. Transients arise in a transmission line when, mainly because of capacitance effects, all frequencies do not suffer the same attenuation (i.e. reduction in amplitude) in transmission, nor the same change in phase, and under these conditions the line does not immediately respond to any change in the electrical conditions at the sending end but undergoes a transitory condition when passing from one steady state to another. In telephony it is usual to design the line and repeater equipment so that attenuation distortion and phase distortion are negligible, and in general therefore transient conditions are not of importance. In the telegraph case this is neither necessary nor economically practicable; in fact one of the features of telegraphy is that perfect reproduction of the transmitted intelligence is possible on lines which, due to the electrical distortion, would be wholly incapable of any other form of transmission. Direct current telegraph transmission problems are therefore almost entirely concerned with transient conditions, and because the mathematical solution of transient phenomena is very complex no easily applicable formulae are available for practical use. The transmission technique is therefore largely based on practical

9

experience supplemented by a knowledge of the electrical pheno-
mena, derived from mathematical and experimental studies.

Certain telegraph transmission problems can be solved, and
considerable help is given in studying most problems, by what
is known as the "steady-state" solution of electrical trans-
mission. This mathematical treatment is concerned with the
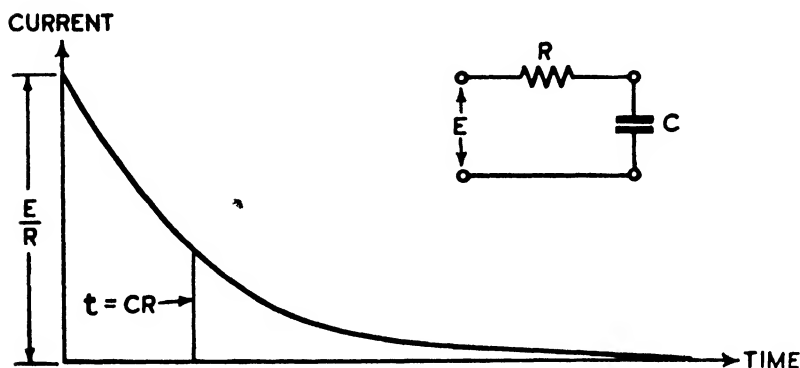
FIG. 4. CURVE OF CHARGING OR DISCHARGING CURRENT
FOR CONDENSER AND RESISTANCE

response of the system to a sinusoidal alternating e.m.f. after
all transient conditions have subsided and provides the well-
known formulae used in telephone transmission. Some account
of steady-state transmission theory in relation to telegraph
transmission will be given later.

Before passing to the more formal discussion of direct
current telegraph transmission some account will be given of
certain simple transient conditions.

Charge and Discharge of a Condenser. When a battery is
connected to a non-inductive resistance the current immedi-
ately assumes a value E/R where E is the voltage across the
resistance and R the value of the resistance. In other words
there is no transient effect. If a condenser is joined in series
with the resistance as shown in Fig. 4, the current at first
assumes the value of E/R but then gradually dies away as
shown in the graph. This is because there is no charge on
the condenser initially, and therefore the whole of the volt-
age E appears across the resistance. As the condenser becomes
charged, the voltage across the resistance is reduced, and the
current is also correspondingly reduced until finally it is sen-
sibly equal to zero.

The equation for the current at any instant can be obtained by a simple application of the Differential Calculus and is given by

$$*I = (E/R)e^{-t/CR}$$

where I = the current in amperes;

E = the applied voltage;

R = the resistance in ohms;

C = the capacitance in farads;

t = the time from switching on in seconds

and e = the base of Napierian Logarithms and is equal to 2·7183 approx.
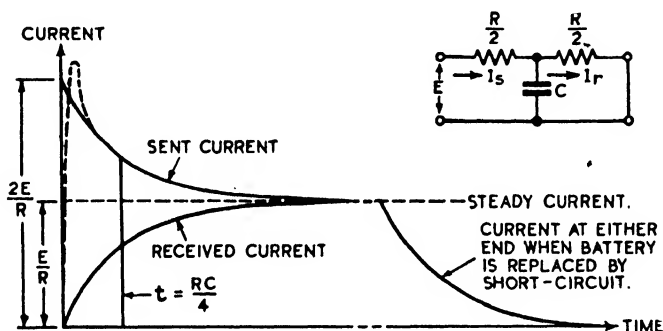
FIG. 5. CURVES OF SENT AND RECEIVED CURRENT
FOR SHORT ARTIFICIAL LINE

(The dotted curve shows the effect of the distributed capacitance near the sending end and the small amount of inductance which would be present in a real line.)

When $t = 0$, that is, at the instant of switching on, $I = E/R$, and when t is great, I is very small. The product CR is called the *time constant* of the circuit, and when $t = CR$ the current has a value $1/2·7183$ times the initial value.

If, when the condenser is charged, the battery is replaced by a short circuit, the condenser discharges through the resistance and the equation for the current is the same as that already given for the charging current.

Single Section Artificial Line. The Tee-network shown in Fig. 5 is a single section artificial line representing a real line of negligible inductance and leakance, having a resistance of R ohms and a total capacitance to earth of C farads. For

* The mathematical derivation of this and subsequent formulae will be found in the Appendix. See also note on Mathematical Notations on page 104.

the artificial line the capacitance is assumed to be lumped at the centre, and this is quite legitimate providing the length of line represented is not too long. Such a network could be used to represent lengths of normal underground telegraph line of up to about 20 miles at the important frequencies present in direct current telegraph signals.

Assume that a voltage E is applied at one end of the line and the other end is earthed. It is easy to see that the initial current in the first resistance is given by $I_s = 2E/R$, there being initially no potential across the condenser. Similarly the initial received current will be zero. As the condenser charges, the value of I_s will diminish while I_r will increase until finally, when the condenser is charged, the currents in each resistance will be steady and equal to E/R, while the condenser will be charged to a voltage $E/2$.

The equations for the currents are

$$I_s = \frac{E}{R}\left(1 + e^{-(4/RC)t}\right)$$

$$I_r = \frac{E}{R}\left(1 - e^{-(4/RC)t}\right).$$

In this case the time constant is $RC/4$ and the time required for the received current to build up to a given proportion of the final value is proportional to the product of the resistance and capacitance of the line.

If the line is now short-circuited at the sending end, the condenser discharges, the discharge current at each end being given by

$$I = \frac{E}{R}e^{-(4/RC)t}.$$

The curves given in Fig. 5 illustrate the foregoing effects.

If the voltage is applied to the line through a resistance of x ohms and the line is terminated by a resistance of y ohms, the formula for the received current becomes

$$I_r = \frac{E}{R + x + y}\left(1 - e^{-\frac{4(R + x + y)}{CR[R + 2(x+y) + (4xy/R)]}t}\right)$$

and the time constant is

$$\frac{CR}{4}\left(1 + \frac{x + y + (4xy/R)}{R + x + y}\right)$$

This shows that the time constant is increased by the presence of resistance at either the sending or receiving end of the line, and therefore the inclusion of resistance will in each case increase the time taken for the current to reach the steady value. It should be noted that as the equation is symmetrical in x and y, the effect of a given resistance is the same whether it is at the receiving or sending end.

The resistance of a 20-mile length of a 20 lb single-wire
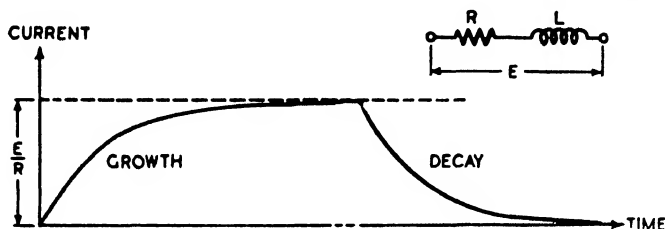
Fig. 6. Curves of Current in an Inductive Circuit

underground circuit is 880 Ω, and the capacitance to earth 2 μF; the time constant with no external resistance is therefore $(880 \times 2 \times 10^{-6}/4) = 0.44$ msec.

The current would be within 5 per cent of the steady value after 1·4 msec. If a battery resistance of 300 Ω is used and the resistance at the receiving end is 3 000 Ω, these times become 1·2 and 3·9 msec respectively.

Growth and Decay of Current in an Inductive Circuit. Similar expressions to the foregoing are obtained for the current in a circuit containing resistance and inductance. In the case shown in Fig. 6, on switching on the applied e.m.f. the current is prevented from rising immediately to the final value by the opposing e.m.f. created in the inductance. This opposing e.m.f. is at all times proportional to the rate of change of the current.

The equation for the current is

$$I = \frac{E}{R}(1 - e^{-(R/L)t})$$

where L = inductance in henries.

When the battery is replaced by a short-circuit, the current falls gradually to zero owing to the e.m.f. induced in the inductance. Thus, just as the energy stored in the dielectric is dissipated in the circuit when a condenser is discharged,

so the energy stored in the magnetic field is dissipated in the circuit when this field collapses on removal of the applied e.m.f., the expression for the current being of the same form in each case.

The current in the present case is given by

$$I = \frac{E}{R} e^{-(R/L)t}.$$

The time constant for the inductive circuit is L/R, and for
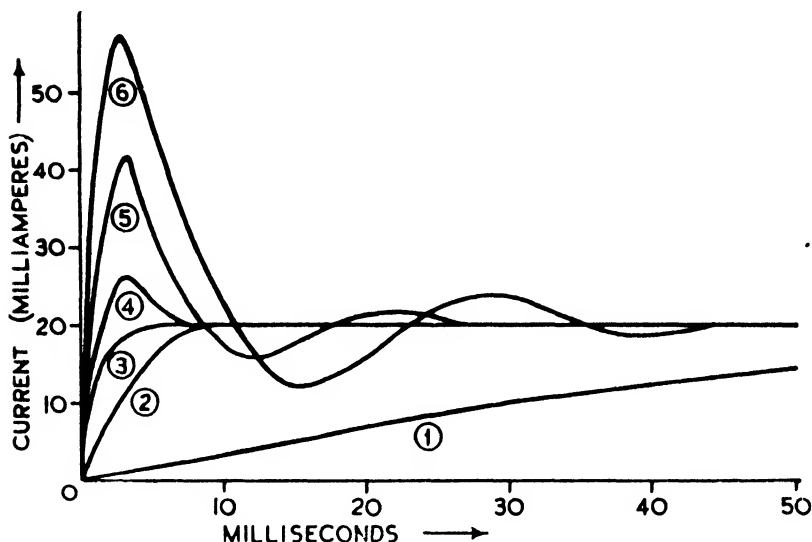
FIG. 7. SHUNTED CONDENSER: EFFECT OF
VARYING CAPACITANCE

a given inductance this can be reduced by increasing the value of R. If R and E are increased in the same ratio, the final value of the current remains constant. It is therefore possible to reduce the time of operation of a relay by adding resistance in series, and at the same time increasing the applied voltage so as to maintain the value of the current within the operating range of the relay.

Shunted Condenser in Relay or Electro-magnet Circuits. The effect just mentioned can be still further enhanced by shunting the additional resistance by a condenser. By this means, because at the outset the condenser is uncharged, a higher voltage is applied to the inductance so still further increasing the rate of growth of the current. Shunted condensers

are for this reason included in the operating circuits of the electromagnets of printing telegraph receivers.

Shunted condensers are also used at the ends of long telegraph lines to correct the distorting effects of the line: their use in this connexion involves quite different factors however and will be considered separately at a later stage.

The mathematical solution of the case where the shunted condenser is used on a short line or in the local circuit of a relay is not difficult, but the equations obtained are cumbersome and of very little practical value, and have therefore been excluded from the following treatment.

The values of the resistance and capacitance to be used in any given case are mainly dependent on the voltage to be used, the operating current of the relay, or electromagnet and its inductance. They are also to some extent dependent on the way in which the inductance varies with current, the speed at which the circuit is to be operated, and the mechanical design of the relay or electromagnet.

In general, the voltage will be fixed by considerations of the source of supply and conditions of use. It will be clear from the remarks relating to the growth of current in an inductive circuit that improved performance will be obtained by using a high rather than a low voltage. An 80 V double current supply has, for a variety of reasons, been fixed as most suitable for the purposes of the British Post Office. The voltage being fixed, it immediately follows that the resistance should be high enough to limit the current to a value consistent with the sensitivity of the relay or electromagnet. The capacitance will then be adjusted to secure the desired increase in the rate of growth of current.

The effect produced by the condenser is illustrated in Fig. 7, which relates to an electromagnet having an inductance of 6 H. The conditions are given in the following table.

Curve	Voltage	Series Resistance (Ω)	Capacitance (μF)
1	6	nil	nil
2	80	4 000	nil
3	80	4 000	0·2
4	80	4 000	0·5
5	80	4 000	1·0
6	80	4 000	2·0

The effect of the addition of resistance, the voltage being increased to maintain the same steady current, is clearly illustrated by curves 1 and 2. The effect of the condenser is illustrated by curves 3 to 6. At a certain stage an oscillation due to the interaction of the capacitance and inductance is produced, causing the current to build up to a value in excess of the steady current. The amplitude of this oscillation is increased as the capacitance is increased, and if it is too great it may cause undesirable signal distortion by interfering with the next operation of the electromagnet or relay. It follows that on high-speed circuits less oscillation is permissible than on low-speed circuits.

Curves similar to 3 and 4 are suitable for the operation of telegraph relays in local circuits. A relay such as the Western Electric Permalloy Relay (P.O. No. 299) which has anti-chatter springs and a light movement, should not have too violent operation or contact chatter results, and best results are sometimes obtained with resistance only. Other relays such as the P.O. Standard Relay give best operation with more "overshoot" such as in curve 4. In the case of the teleprinter where the armature of the electromagnet carries an appreciable mechanical load, overshoot approaching that shown in curve 5 is beneficial so as to supply the energy necessary to obtain a quick transit of the armature between the stops.

In general the best arrangement for any given case is found by experiment, and then standardized.

For teleprinters 3 and 7 the values are 4 000 Ω and 4 μF using a double current 80 V supply, but in this case the electromagnet is also shunted by a 2 μF condenser, and, although this condenser tends to slow down the build-up of the current in the coils of the electromagnet, this effect is more than compensated by the effect of the large value shunted condenser chosen. This extra condenser was originally added because some improvement was obtained when working over long lines, but its use is now retained mainly because it suppresses the harmonics produced in the coils of the electromagnet which would otherwise pass out to line and cause interference on telephone circuits working in the same cable.

The Arrival Curve. The whole of the effects considered in the preceding section were the results of transient conditions set up by the presence of capacitance or inductance in the electric circuit preventing the current immediately assuming its final steady value. The transmission efficiency of a long

direct current telegraph line is dependent on the extent of the transient condition produced in a similar manner by the capacitance of the line. The nature of these transients can be determined from the *arrival curve*.

An arrival curve is a curve showing the building up of the current received at the end of a transmission line when a steady voltage of known value is suddenly applied at the sending end, the line being previously in a state of rest, and the sending and receiving impedances being specified. The shape of the
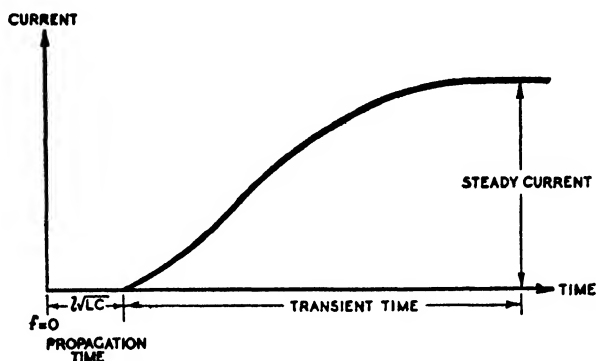
FIG. 8. ARRIVAL CURVE FOR A TYPICAL UNLOADED LINE

arrival curve determines the response of the system to an applied voltage of any character whatever. Where transient conditions are not of importance, the steady-state response of the line to sinusoidal voltage can be used to determine the performance, but in telegraphy the study of the transmission phenomena from the point of view of the arrival curve is very valuable. Lord Kelvin was first responsible for this method of treating telegraph transmission and the publication in 1855 of his mathematical researches led to the famous "KR" law.

Fig. 8 shows the arrival curve for a typical unloaded line, and the transient time is indicated. The propagation time is the time taken for the propagation of an electrical impulse through the medium, and is the time which elapses before any effect occurs at the receiving end.) The calculation of the arrival curve in any given case, taking account of the resistance, leakance, inductance, and capacitance of the line and the value of the terminal impedances, is a matter of some complexity, and owing to the considerable amount of labour involved is rarely attempted in practice.

The case of the simple artificial line given earlier, in which only resistance and capacitance are present, can be readily extended to lines having a number of sections.

Lord Kelvin gave the solution in the case of a line without leakance and inductance and when the terminating impedances are zero, as—

$$I = 2\,(E/Rl)\,(\tfrac{1}{2} - \theta + \theta^4 - \theta^9 + \theta^{16} - \ldots)$$

where $\theta = e^{-(\pi^2 t/CRl^2)}$

To a first approximation this may be written

$$I = (E/Rl)\,(1 - 2e^{-(\pi^2 t/CRl^2)} + 2e^{-(4\pi^2 t/CRl^2)})$$

The foregoing equation shows that the time required for the current to rise to a given fraction of the final value is proportional to CRl^2. For example, the current reaches half its final value when $t/CRl^2 = 0.14$ (approx.), or $t = 0.14\ CRl^2$ sec and is almost equal to the steady value for $t = 0.55\ CRL^2$.

On this basis the KR law* may be stated as follows—

On a telegraph line of negligible inductance and leakance with zero terminal impedances the time delay of an electrical operation at the receiving end is proportional to CRl^2.

This relationship between CRl^2 and the build-up time has been used to evaluate empirical formulae of the form

Maximum speed of working $= k/CRl^2$,

where k is a constant dependent upon the system of working and the terminal apparatus and signal shaping networks. Values for k for submarine and underground cables† have been determined from the result of practical experience and used to a certain extent in the past, but in Land Telegraphy, owing to modern developments in the design of relays and telegraph distortion measuring equipment, it is possible to secure performances far in excess of those given formerly and, owing to its limitations, this method of expressing the transmission qualities of a circuit has fallen into disuse. Also, the extensive use of fixed speed systems such as Teleprinter and Multiplex, in which the maximum permissible speed of working the circuit is of no real interest, has rendered the method of still less importance. Nevertheless, although in modern land telegraphy the KR law is of little practical importance,

* K was the former symbol for capacitance.

† Owing to the much lower capacitance the formula is not applicable to overhead lines where conductor and insulation resistance and external interference play a more important part.

an understanding of the underlying principles is of value in studying telegraph transmission problems, and that alone is the reason for its inclusion here.

Owing to the labour and difficulty of calculating the arrival curve accurately in practical cases, in which the value of the terminal impedances are taken into account, the usual practice is to record it by means of an oscillograph,* using an actual line or an artificial line constructed to represent the case in question.

An interesting general solution of the arrival curve in which all the electrical constants of the line and terminal equipment are taken into account has been given by H. W. Malcolm.† This solution makes use of ordinary steady-state transmission theory as used in telephony, and although it, too, involves considerable labour, it is an important aid to an appreciation of the relation between steady-state and transient phenomena without recourse to obstruse mathematics, and for this reason it will be returned to later after some account has been given of steady-state transmission theory.

Construction of Telegraph Signals from the Arrival Curve. Principle of Superposition Theory. It is possible by use of the method of superposition, employed by Heaviside in his work on transmission problems, to construct the signals received at the end of a line from a knowledge of the arrival curve. In this theory it is assumed that the wave-form of the applied voltage consists of a large number of sudden small increases or decreases in voltage. Each one of the increases is supposed

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* An *oscillograph* is an instrument used for obtaining a visual or photographic record of the current in a circuit in the form of a curve showing variations in amplitude plotted against time, such as the signal shapes shown in Fig. 2. In an electromagnetic oscillograph the current indicator is in effect a moving coil galvanometer in which the coil consists of a single loop of fine wire which carries a small mirror, the whole being usually suspended in oil to damp out mechanical resonance. An intense beam of light is focused on the mirror and reflected as a small spot on to a strip of photographic paper mounted on a drum. To obtain a record the drum is rotated, the position of the spot at any time being determined by the amplitude of the deflection of the coil which in turn depends on the value of the current. Alternatively the beam of light is reflected from a system of rotating mirrors on to a translucent screen. The rotation of the mirrors causes the spot of light to traverse the screen and so give a picture of the wave shape under examination.

In a *cathode-ray oscillograph* a stream of electrons in a cathode-ray tube is deflected by the current or voltage concerned and a picture is secured by arranging for the beam to impinge on a fluorescent screen deposited on the end of the glass container.

† *The Theory of the Submarine Telegraph and Telephone Cable.* H. W. Malcolm, London, 1917.

· to produce a received current corresponding to the arrival curve, and each decrease—being assumed to be due to the sudden insertion of a voltage in the opposite sense—is supposed to produce therefore an arrival curve of the opposite sense to that for an increase. The summation of the resultant series of arrival curves gives the shape of the received current, providing the values of the resistance, inductance, capacitance and leakance in the circuit are independent of the value and rate of change of the current.)

In telegraphy the process is simplified because the signals merely consist of sudden changes or reversals of voltage, and therefore the signal shapes may be constructed fairly readily by graphical means. This is illustrated in Fig. 9 for three simple double-current signals. It should be noted that for double-current signals the arrival curve has twice the amplitude of that for a single-current signal using the same voltage. That is, the arrival curve for the reversal of a voltage V, is the same as that for switching on a voltage $2V$ except that the zero current position is displaced as shown in Fig. 9 owing to the presence of the steady-current in the line at the commencement.

In the figure the received signal (a) was constructed from the arrival curve due to the first reversal, and the arrival curve due to the second reversal (shown dotted and labelled (a)). To avoid confusion this has been drawn at the bottom of the figure, but should, strictly speaking, commence at the zero current line. The ordinates for this curve are subtracted from those for the first curve to obtain the resultant signal. When both arrival curves have reached the steady-state, the current due to the first is $+ I$ (after allowing for the steady current $- I$ which existed at the outset), and to the second $- 2I$, and the resultant current is therefore $- I$, that is, the original steady-state value. The figure also shows the effect on the received signal of the relation of the length of the signal to the build-up time of the arrival curve. Signal (a) has a duration less than 'one-quarter of the build-up time, and the received current never actually reverses in direction. The process is further illustrated in Fig. 10, in which the series of arrival curves due to square-topped voltage reversals impressed at the sending end of a line is shown. T is the periodic time of the reversals, and they are assumed to be switched on at time $t = 0$ and switched off at time $t = 9T/2$. The received current as given by summing the arrival curves is also shown, and it
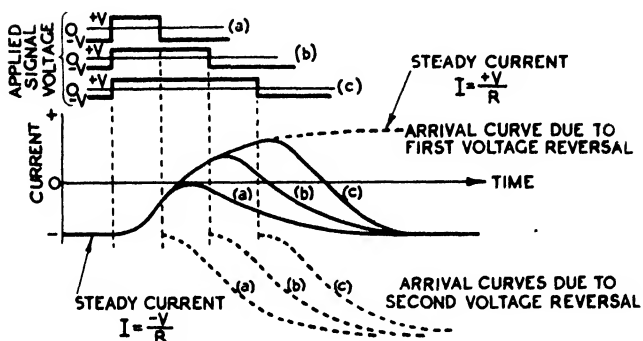
FIG. 9. CONSTRUCTED SIGNAL SHAPES FOR THREE SIMPLE SIGNALS
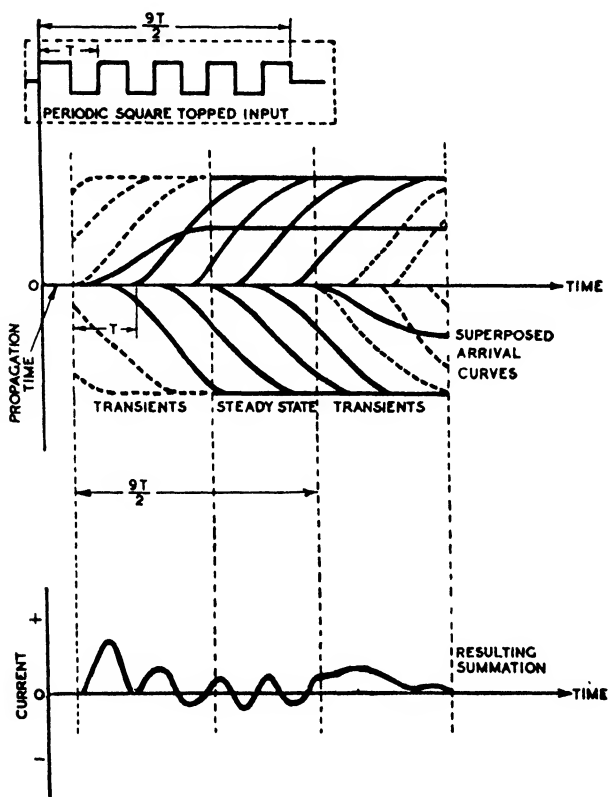
FIG. 10. ARRIVAL CURVES DUE TO SQUARE-TOPPED VOLTAGE REVERSALS

3 — (T.444)

will be seen that not until the first arrival curve has reached the steady value does the received current settle down to symmetrical alternations. This period corresponds to the transient time. On switching off, transients again occur and persist for a like period, the total period during which the current persists in the line being equal to the time of application of the signals plus one transient period. Transients will occur to a greater or less degree in all circuits unless the circuits are completely distortionless, in which case the arrival curve is rectangular and the transient time is obviously zero.

Single-Current Signals. If in single-current working the line is looped or earthed at the sending end during intervals of no current through a resistance equal to that of the battery circuit, the signal shapes will be exactly the same as those obtained in double-current working. This effect would be secured in the circuit shown in Fig. 2 if the negative battery were removed and the resistance coil connected directly to earth. The only change in the graphs relating to line current in this figure, if these were then for single-current with a single positive battery, would be that the zero current line would be shifted to the position corresponding to that of the steady negative current in the double-current case. It should be noted that a reversal of current occurs at the sending end, as will be seen by referring to the curve of sending current in Fig. 2. This is due to the surge of discharge current on reverting to the no-current conditions. That the single current signal shapes must be the same as those for double-current working will be apparent if it is supposed that the no-current condition is produced by inserting a battery of equal and opposite voltage to the first battery. Then by the principle of superposition this second battery will produce an arrival curve identical with that due to the first battery but of opposite sense, and the summation of the two curves will give the received signal.

For this method of single-current working, if the receiving relay is biased magnetically or electrically to an extent exactly equivalent to that due to half the steady line current, the operation will be identical with that obtained with double-current working using a voltage half that of the single-current battery. The difficulty of ensuring that at all times the bias of the relay will be equivalent to half the line current explains the superior results obtainable with double-current working.

Closed Circuit Working. This is a system of single current

working used for working between an out office and a head office, and avoids the provision of a line current supply at the out office. A simplified diagram of the arrangement used on the extension circuits to the renter's premises on direct current Tariff A private wires is given in Fig. 11. To illustrate the type of transmission obtained, an artificial line suitable to represent an extension circuit of up to about 20
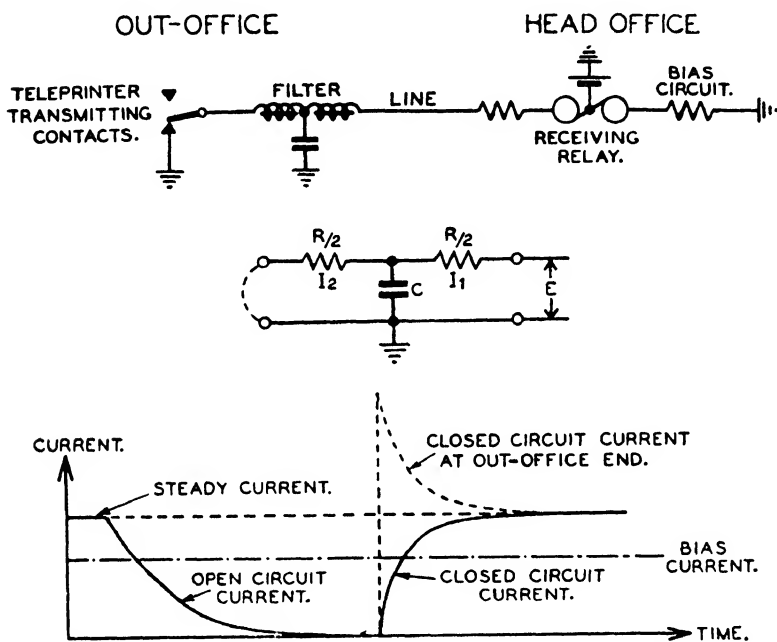
FIG. 11. CLOSED CIRCUIT WORKING ON SEND LEG OF TELEPRINTER PRIVATE WIRE
Local end

miles in length has been taken, the filter and receiving relay having been omitted for simplicity. Transmission is by opening and closing the circuit at the out office.

When the circuit is closed, the current is equal to E/R, and when it is open-circuited the current at the battery end is given by—

$$I_1 = \frac{E}{R} e^{-(2/RC)t}.$$

When the short-circuit is replaced, the current at the battery end is given by—

$$I_1 = \frac{E}{R} (1 - e^{-(4/RC)t}),$$

and that at the other end by—

$$I_2 = \frac{E}{R}(1 + e^{-(4/RC)t}).$$

It is interesting to note that these expressions are the same as those given for the artificial line shown in Fig. 5, except that the expressions for sent and received current have changed over. This is because when the line is short-circuited the condenser has already been charged to the full voltage of the battery during the open-circuit condition.

The curves for the current at the battery end of the circuit are given in Fig. 11. As the time constant for the open circuit case ($RC/2$) is twice that for the closed circuit ($RC/4$), this method of working leads to unbalanced signals, and it is necessary to counteract this by increasing the bias current above the normal value of half the steady line current.

In practice the signal shapes are modified by the presence of the filter, relay, and current-limiting resistance. In particular the condenser in the filter prolongs the discharge through the relay at open circuit, and exaggerates the unbalance effect already mentioned, especially on the longer lines.

The unbalance effect can be reduced by removing some of the current limiting resistance to the out office. Another method used on a similar type of circuit in America is to insert an inductor in the line, so slowing up the build-up of the current to make it more comparable with the die-away.

In the system of working used in the British Post Office, the second wire of the pair is worked double-current from the head office to the teleprinter at the out office; whereas in another system largely used in America, the circuit is used as a loop, the electromagnet and transmitting contacts being connected in series at the out office. In this case at "break" the current in the electromagnet immediately falls to zero, while at "make" the current does not rise rapidly as indicated by the dotted curve in Fig. 11, but in a manner similar to that shown by the full-line curve, owing to the inductance of the electro-magnet.

Aerial Lines: The Effect of Leakage. There are relatively few telegraph circuits of any length working on aerial lines in the British Post Office, and none of sufficient length to cause transient effects to be of importance. The capacitance per mile of an aerial circuit is about one-sixth of that of an underground

circuit, and the resistance is generally lower, because of the heavier gauge conductors used. The time required for the current to rise to 90 per cent of its full value on a 200 mile, 150 lb, copper line with resistances of 150 Ω at each end is less than 2 msec, so that even at high telegraph speeds the transient effects due to the line are unimportant. Of much more consequence are the effects of leakage and external interference. The result of leakage is to reduce the value of the current available at the receiving end. As a limit must be placed on the voltage applied to line, the length of circuit which can be worked without the use of repeaters is largely dependent on the value at which the insulation resistance of the line can reasonably be expected to be maintained, bearing in mind climatic and other relevant factors. The actual effect of the leakage is dependent on whether it is approximately uniformly distributed, or due to abnormal conditions at one or more localized points. Both cases will be dealt with.

Uniformly Distributed Leakage. The effect of leakage in gradually reducing the value of the transmitted current needs no explanation. What is more interesting is its effect in reducing the time-constant or transient time. As the transient time is in any event quite small, this fact is of little more than theoretical interest in the present case, but nevertheless it will be instructive to examine it in a little detail.

Steady-State Solution. In the first place the solution for the steady current condition will be given. In this condition the capacitance takes no part, simply carrying a steady charge, and the line may be treated as merely having resistance and leakance. In this case it is unnecessary to refer to a ladder network simulating the line in order to simplify the mathematics. The received current is given by the equation—

$$I_r = \frac{E}{(R_t + R_s)\cosh\sqrt{(RG)}.l + [R_o + (R_sR_t/R_o)]\sinh\sqrt{(RG)}.l}$$

where E = applied voltage;

$\quad R$ = conductor resistance per unit length (ohms);

$\quad G$ = leakance per unit length (mhos);

$\quad l$ = length of line;

$\quad R_t$ = terminating resistance (ohms);

$\quad R_s$ = sending end resistance (ohms);

$\quad R_o = \sqrt{(R/G)}$ = "characteristic resistance" of line (ohms);

Cosh $\sqrt{(RG)} . l = \frac{1}{2} [e^{\sqrt{(RG).l}} + e^{-\sqrt{(RG).l}}]$ (hyperbolic cosine) ;*
and

Sinh $\sqrt{(RG)} . l = \frac{1}{2} [e^{\sqrt{(RG).l}} - e^{-\sqrt{(RG).l}}]$ (hyperbolic sine).

The foregoing equation is a particular case of the general solution of the steady-state transmission equation which will be dealt with later. The mathematical proof is given in the Appendix and it should, if not already known, be studied at this stage.

It will be seen that from this equation the received current can be calculated for any terminal conditions.

One special case worthy of mention is when $R_s = 0$ and $R_t = R_o$. Substituting these values in the foregoing equation gives

$$I_r = \frac{E}{R_o [\cosh \sqrt{(RG)} . l + \sinh \sqrt{(RG)} . l]}$$

or $I_r = \dfrac{E}{R_o} e^{-\sqrt{(RG).l}}$

To illustrate the use of the full equation, the case will be taken of an aerial line of 200 miles in length having the following constants—

$R = 6 \, \Omega/\text{mile}.$

$G = 1/60\ 000$ mhos/mile (i.e. insulation resistance

$= 60\ 000 \, \Omega/\text{mile}).$

$R_t = 150 \, \Omega.$

$R_s = 150 \, \Omega.$

We have

$R_o = \sqrt{(R/G)} = 600 \, \Omega.$

$\sqrt{(RG)} = 0.01.$

$I_r = E/(300 \cosh 2 + 637.5 \sinh 2).$

The values of cosh and sinh can be obtained from tables of hyperbolic functions and substituting these values we have

$I_r = E/(300 \times 3.762 + 637.5 \times 3.626)$

$= E/3\ 438.$

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* The hyperbolic functions which appear in this and other similar equations given later, if they have not been met before, should not be regarded as having any mysterious significance and need only be looked upon as a convenient method of writing and handling the particular combinations of the exponential terms. See also footnote on page 113 and Fig. 18.

If the line has ·no leakance the current at all points in the circuit is given by

$$I = E/(R_s + R_l + R_t) = E/1\ 500.$$

The effect of the leakage is to reduce therefore, the received current to a value 0·437 of the current without leakage. The corresponding figure for an insulation resistance of 200 000 Ω/mile is 0·765.

The formula shows that the value to which the received
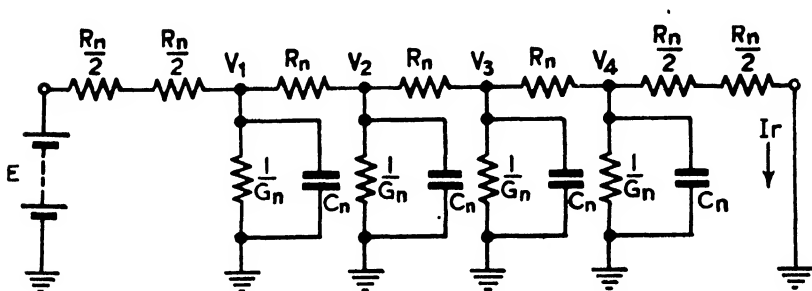
FIG. 12. ARTIFICIAL LINE WITH LEAKANCE AND CAPACITANCE

current is reduced is dependent on $\sqrt{(RG)}.l$, and therefore when the conductor resistance per mile is high the effect of leakage is more serious. Again, the greater the length of the line the more serious the effect of leakage. For a 500 mile length of line the same as the foregoing, the ratios quoted would be 0·022 and 0·14 for 60 000 and 200 000 Ω/mile insulation resistance respectively.

Transient Solution. To obtain a general solution for the growth of current when a battery is suddenly applied to the line is a matter of some mathematical complexity, but the case under consideration can be readily solved by recourse to the artificial line method.

In Fig. 12 is shown a network of resistors and condensers which can, by suitable choice of components, be used to represent for all practical purposes an aerial line of some 200 miles in length, each section representing 50 miles.

The line is assumed to be connected to terminal resistances of $R_n/2$ ohms, where R_n is the resistance of 50 miles of conductor. This value has been chosen merely to simplify the mathematical solution. The equation for the received current when a battery of E volts is applied at the sending end and the leakage and inductance is negligible is

$$I_r = \frac{E}{5R_n}\,(1 - \frac{5}{5 - \sqrt{5}}\,e^{-\frac{1}{2}(3 - \sqrt{5}).t/C_nR_n}$$

$$+ \frac{1}{3 - \sqrt{5}}\,e^{-\frac{1}{2}(5 - \sqrt{5}).t/C_nR_n} - \frac{5}{5 + \sqrt{5}}\,e^{-\frac{1}{2}(3 + \sqrt{5}).t/C_nR_n}$$

$$+ \frac{1}{3 + \sqrt{5}}\,e^{-\frac{1}{2}(5 + \sqrt{5}).t/C_nR_n})$$

where R_n = conductor resistance of each section;

C_n = capacitance of each section.

When leakance is included the formula becomes

$$I_r = \frac{E}{R_n a.b.c.d}\,(1 - \frac{b.c.d}{5 + \sqrt{5}}\,e^{-a(t/C_nR_n)} + \frac{a.c.d}{5 - \sqrt{5}}\,e^{-b(t/C_nR_n)}$$

$$- \frac{a.b.d}{5 - \sqrt{5}}\,e^{-c(t/C_nR_n)} + \frac{a.b.c}{5 + \sqrt{5}}\,e^{-d(t/C_nR_n)})$$

where $a = R_nG_n + \frac{1}{2}(3 - \sqrt{5})$;

$b = R_nG_n + \frac{1}{2}(5 - \sqrt{5})$;

$c = R_nG_n + \frac{1}{2}(3 + \sqrt{5})$;

$d = R_nG_n + \frac{1}{2}(5 + \sqrt{5})$;

and G_n = the leakance per section.

(*Note.* $R_nG_n = RG$, and $C_nR_n = CRl^2/16$ where R, G, and C are the values per mile.)

It will be seen that the time constant of each of the exponential terms has been decreased by the introduction of the leakage factor; for example that for the first term is

$$C_nR_n/\tfrac{1}{2}(3 - \sqrt{5})$$

without leakage, and $C_nR_n/[R_nG_n + \frac{1}{2}(3 - \sqrt{5})]$ when leakage is present. The effect of the leakage is therefore to reduce the duration of the transient effects and improve the received signal shapes. It is well known from alternating current transmission theory that leakance reduces the distortion due to the capacitance of the line, and the above simple example confirms this fact.

Although the transient time is reduced, the value of received current is also reduced in the ratio of $(a.b.c.d)/5$. Both effects are dependent on the product $R \times G$. To make the effects more clear, the results will be given for the same case as before,

that of an aerial line, 200 miles in length, having a conductor resistance of 6 Ω/mile and a capacitance of 0·015 μF/mile. The insulation resistance will first be taken as infinity, and then 60 000 Ω/mile. For the 50 miles sections therefore—

$$R_n = 300\ \Omega\,;$$
$$C_n = 0\cdot75\ \mu\mathrm{F}\,;$$
$$G_n = 1/1\ 200\ \text{mho}\,;$$
$$R_n G_n = 0\cdot25.$$

In the first case

$$I_r = \frac{E}{300 \times 5}\,(1 - 1\cdot81\,e^{-10^3 t/0\cdot59} + 1\cdot31\,e^{-10^3 t/0\cdot16}$$

$$-\ 0\cdot69\,e^{-10^3 t/0\cdot09} + 0\cdot19\,e^{-10^3 t/0\cdot06})$$

and in the second case

$$I_r = \frac{E}{300 \times 11\cdot43}\,(1 - 2\cdot5\,e^{-10^3 t/0\cdot36} + 2\cdot53\,e^{-10^3 t/0\cdot14}$$

$$-\ 1\cdot44\,e^{-10^3 t/0\cdot08} + 0\cdot41\,e^{-10^3 t/0\cdot06}).$$

The curves given by these equations are shown in Fig. 13. To show the comparative shapes of the curves the dotted curve has been drawn to represent the case where the applied voltage has been increased in the ratio 11·43/5 to compensate for the leakage current. The improved shape is at once apparent.

The steady current received when leakage is present is 5/11·43 = 0·437 times the current without leakage, and it should be noted that this ratio is the same as that given in the preceding section when it was obtained by the use of the general steady-state formula.

Effect of Low Insulation Resistance Faults. The value of insulation resistance of 60 000 Ω/mile was taken in the preceding paragraphs as being the lowest value tolerated in this country under the most adverse conditions. A minimum value of 200 000 Ω/mile is the standard aimed at under normal conditions. Where the loss of current is mainly due to a localized fault or bad section of line, its seriousness will depend on its position.

When the insulation resistance is high compared to the fault resistance the received current is given by

$$I_r = EF/[RR_1 + F(R + R_1)]$$
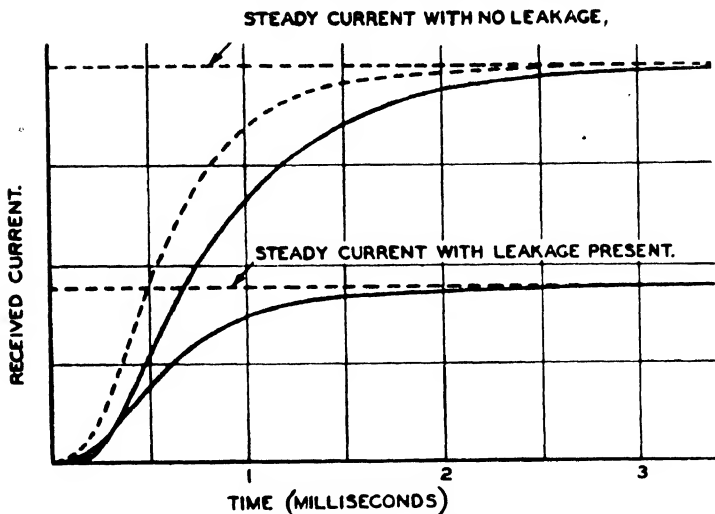
FIG. 13. ARRIVAL CURVES FOR AERIAL LINE WITH AND WITHOUT LEAKANCE

The dotted curve represents the arrival curve with leakance, but having the ordinates increased in the ratio 11·43/5.
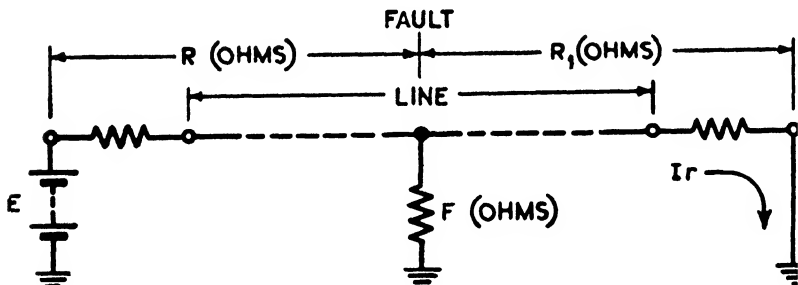
FIG. 14. LINE WITH LOCALIZED INSULATION RESISTANCE FAULT

where E = applied voltage;

R = resistance to the fault (including terminal resistance);

R_1 = resistance beyond fault (including terminal resistance); and

F = resistance of fault.

The conditions are illustrated in Fig. 14. The only factor depending on the position of the fault is $R R_1$, which is a maximum, and therefore the current a minimum, when $R = R_1$, that is, when the resistance each side of the fault is

the same, RR_1 is a minimum, and therefore the current a maximum, when R or R_1 is smallest, that is, when the fault is at the end of the circuit where the terminal resistance is smallest. When both terminating resistances are the same, as would probably be the case for a duplex circuit, the received current is a minimum when the fault is at the centre of the line. On a simplex circuit where the receiving resistance may be much higher than the sending resistance, the current will be a maximum when the fault is at the sending end, and the point for minimum received current will be nearer the receiving end. If the receiving resistance is equal to or greater than the sum of the line and sending resistance, the received current will be a minimum when the fault is at the receiving end of the line.

Alternating Current Transmission Theory : General. In the preceding section the transmission theory given was mainly concerned with transient conditions. Ordinary a.c. transmission theory is, on the other hand, mainly concerned with the steady-state conditions existing when an alternating e.m.f. which is a repeating function of time, is impressed on a transmission line, and all transient conditions have subsided. In telephony and a.c. telegraphy the transmission links are usually designed so that transient conditions are reduced to negligible importance, and the steady-state response of a circuit is therefore, in general, an adequate guide to its performance.

The fact that only steady-state conditions are to be considered involves the condition that the applied e.m.f. must be a periodic function of time, that is to say, that any variations in amplitude which the e.m.f. undergoes must be cyclic in character. It is well known from Fourier's Theorem that any periodic function can be represented as the sum of a series of simple sinusoidal functions of suitable amplitude, phase, and period. Thus, within certain limits, the response of a transmission line to a complex periodic e.m.f. may be examined by determining its steady-state response to sine wave e.m.f.'s of suitable frequency. In order therefore to establish the performance of the line, it is only necessary to know its response to those frequencies which will be contained in the signals or wave forms it is desired to transmit. This fact forms the basis of alternating-current transmission theory as used in telephony and a.c. telegraphy. Furthermore, the assumption of a sine wave e.m.f. considerably simplifies the mathematics.

Simple Sine Function. The equation

$$e = E \sin \omega t \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (1)$$

relates to a simple harmonic or sine wave where

> e is the amplitude at any instant;
>
> E is the maximum amplitude of the wave;
>
> t is the time;

and ω is a constant depending upon the frequency with which the wave repeats itself.

The curve of the foregoing equation is plotted in Fig. 15, the ordinates giving the amplitude of the wave or e.m.f. at any given time from the commencement of a cycle or period. The relation of the curve to a sine function is illustrated by means of the rotating vector shown at the left. One complete rotation of the vector corresponds to one cycle of the sine wave. The figure shows how, for a given fraction of a revolu-
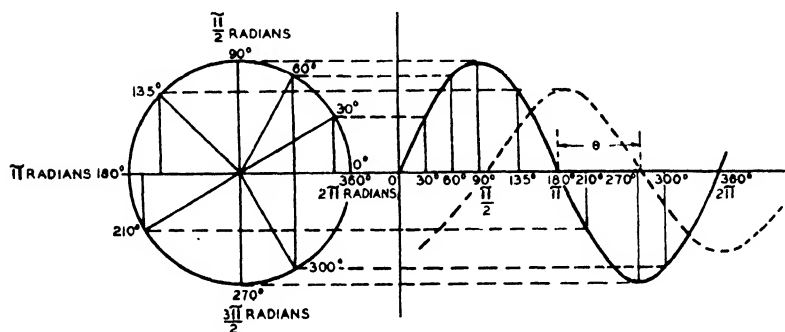
FIG. 15. SINE CURVES
$E \sin \omega t$ and $E_1 \sin (\omega t - \theta)$

tion of the vector, 30° for example, the amplitude of the sine wave for the same fraction of its total period, is equal to the sine of the angle turned through by the vector. The vector is assumed to rotate in a counter-clockwise direction. It will be seen that any point on the sine wave corresponds to a given angular displacement of the rotating vector. This angle is reckoned in radians and in equation (1) this is given by the product ωt. Thus when ωt is equal to $\pi/2$ radians we have—

$$e = E$$

because $\sin \pi/2 = + 1$. Also when $\omega t = \pi$

$$e = 0$$

because $\sin \pi = 0$. When $\omega t = 3\pi/2$

$$e = - E$$

because $\sin (3\pi/2) = - 1$, indicating that the e.m.f. has changed in direction. When $\omega t = 2\pi$ the cycle is completed and a new cycle commences.

It will thus be seen that

$$t = 2\pi/\omega$$

is the period of one cycle, t being the time in seconds.

Again, if f is the frequency, that is, the number of cycles per second, we have

$$f = 1/t = \omega/2\pi, \text{ or } \omega = 2\pi f.$$

In Fig. 15 a second sine curve is shown dotted. This curve
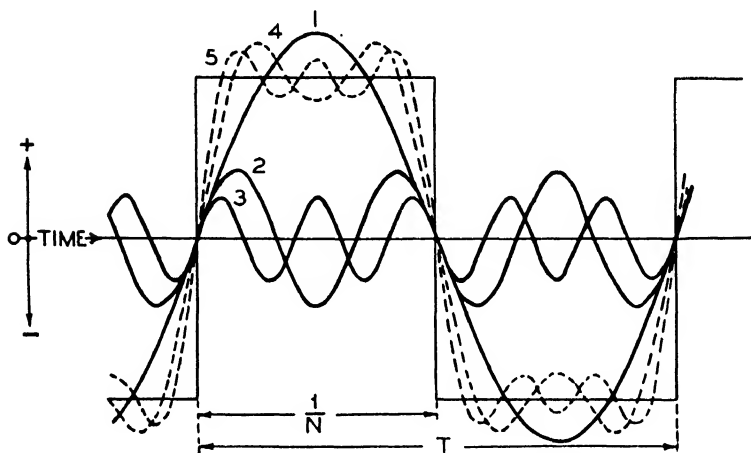
FIG. 16. SINE WAVE COMPONENTS OF SQUARE-TOPPED REVERSALS

is lagging behind the first by an angle θ, or a time of θ/ω sec. The curves are said to have a *phase difference* of θ radians. The equation to the second curve is

$$e_1 = E_1 \sin (\omega t - \theta)$$

where the time t is reckoned from the zero point on the first wave $e = E \sin \omega t$.

Fourier Analysis of Periodic Signals. By way of example as to how periodic signals or waves may be resolved into a series of sine curves, the case of ordinary square-topped voltage reversals will be taken. This is illustrated in Fig. 16.

The Fourier series corresponding to periodic reversals of amplitude E is given by

$$y = (4E/\pi) (\sin \omega t + \tfrac{1}{3} \sin 3wt + \tfrac{1}{5} \sin 5 \omega t$$
$$+ \tfrac{1}{7} \sin 7 \omega t + \tfrac{1}{9} \sin 9 \omega t + \text{etc.}) \quad . \quad . \quad (2)$$

where $\omega = 2\pi/T = 2\pi f$

$\quad\quad T =$ the period in seconds of one complete cycle
$\quad\quad\quad\quad$ and
$\quad\quad f =$ the number of cycles per second.

The first three terms of this series are plotted as curves 1, 2, and 3 of Fig. 16. The first term represents the fundamental frequency of the wave and has the same frequency as the reversals. The remainder of the terms are the odd harmonics of this fundamental.

The accurate representation of the reversals would require these terms to be continued to infinity. Curve 4 of Fig. 16 is the sum of the first two terms, and Curve 5 is the sum of the first three. It will be seen how the resultant approximates more nearly to the square-topped curve as additional terms are included.

It should be noted that the number of signal elements per second (or bauds) is equal to twice the frequency of the fundamental sine wave. Thus—

if N = telegraph speed in bauds,

we have

$$1/N = \tfrac{1}{2}\,T \text{ and } N = 2\,f,$$

where T and f have the same significance as before.

When reversals are transmitted over a long telegraph line, owing to the losses suffered in the line by the higher frequencies these are received at such a small amplitude that only the fundamental frequency remains, and the reversals are received as almost a pure sine wave. This is illustrated in Fig. 10 in which it will be seen that during the steady-state condition the received current is approximately sinusoidal.

To reproduce the square-topped signals faithfully at the receiving end would necessitate the transmission of all frequencies up to infinity. Such reproduction is, however, unnecessary. In practice all frequencies except the fundamental can be eliminated without detriment to the efficiency of the telegraph transmission.

All periodic signals can similarly be resolved into sine wave components, the fundamental frequency always being equal to the number of times the periodic cycle is repeated per second.

It is apparent therefore how the solution of the behaviour of a line in response to sinusoidal wave-forms is a legitimate method of attack to obtain a knowledge of the response to more complicated wave forms.

THE CLASSICAL A.C. TRANSMISSION EQUATIONS

The mathematical problem in a.c. transmission theory is that of finding expressions for the voltage and current at any

point in the line, under given conditions of terminal impedance, when a sinusoidal e.m.f. is applied at one end of the circuit. As this is intended to be a practical rather than a mathematical development of the subject, proofs of the various formulae will not be given here. The derivation of some of the equations —necessarily in a condensed form—has, however, been given in the Appendix, and this would repay examination at this juncture.

In Fig. 17 a single-wire and a loop circuit are represented.
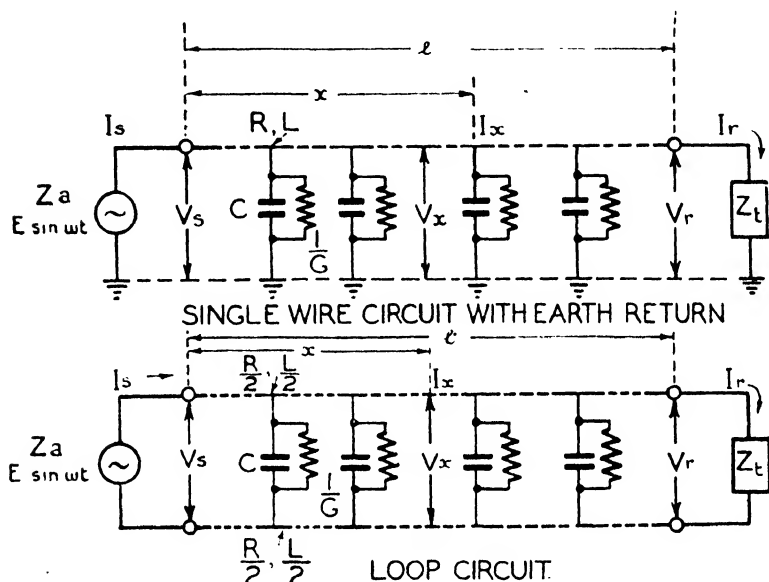
FIG. 17. DIAGRAMMATIC REPRESENTATION OF TRANSMISSION CIRCUITS

The circuits are assumed to be uniform throughout their length as regards their electrical characteristics. By reason of its physical configuration, the line is inherently possessed of four electrical characteristics, called the *primary constants*, viz. resistance, inductance, capacitance, and leakance. The resistance and leakance give rise to loss of the transmitted power by transformation into heat, while the inductance and capacitance, or rather the electromagnetic and electrostatic fields associated with the signal currents, take part in the propagation of the electric waves.

The fundamental a.c. transmission equations derived from

a study of the response of the line to an impressed sinusoidal e.m.f. are

$$\left.\begin{array}{l} V_x = V_s \cosh px - I_s Z_0 \sinh px \\ I_x = I_s \cosh px - (V_s/Z_0) \sinh px \end{array}\right\}* \qquad . \qquad . \qquad . \quad (3)$$

where V_x, I_x = voltage and current (sinusoidal) at any point distance x from the sending end;

$\quad\quad V_s, I_s$ = sending end voltage and current;

$\quad\quad p = \sqrt{[(R + j\omega L)(G + j\omega C)]}$ and is called the *propagation constant* of the line;

$\quad\quad Z_0 = \sqrt{\dfrac{(R + j\omega L)}{(G + j\omega C)}}$ and is called the *characteristic impedance* of the line

R, L, G, C = primary constants per unit length;

$\quad\quad \omega = 2\pi f$, where f is frequency of impressed sine wave;

$\quad\quad j = \sqrt{-1}$.

From equations (3) can be calculated the voltage and current at any point in the line for any given terminal conditions. The voltage and current at the receiving end is obtained by putting $x = l$ in equations (3), l being the length of the line, as follows—

$$\left.\begin{array}{l} V_r = V_s \cosh pl - I_s Z_0 \sinh pl \\ I_r = I_s \cosh pl - (V_s/Z_0) \sinh pl \end{array}\right\} \qquad . \qquad . \qquad . \quad (4)$$

By introducing the term

Z_t = terminating impedance at end of line and noting that

$$V_r/I_r = Z_t$$

we can, by substituting this value in equations (4), obtain the following

$$I_r = \frac{V_s}{Z_t \cosh pl + Z_0 \sinh pl} \qquad . \qquad . \qquad . \qquad .. \qquad . \quad (5)$$

$$I_s = \frac{V_s}{Z_0} \cdot \frac{Z_0 \cosh pl + Z_t \sinh pl}{Z_t \cosh pl + Z_0 \sinh pl} \qquad . \qquad . \qquad . \qquad . \quad (6)$$

* See footnotes *re* Hyperbolic functions on p. 26 and p. 113 of Appendix. Also Fig. 18.

Equation (5) gives the value of the received current in terms of the sending voltage and the terminating conditions of the line. If we introduce the further terms

$$Z_a = \text{impedance of source};$$
$$E = \text{e.m.f. of source}$$

we have

$$V_s = E - I_s Z_a$$

and therefore

$$I_r = \frac{E}{(Z_a + Z_t)\cosh pl + [Z_0 + (Z_a Z_t / Z_0)]\sinh pl} \qquad . \quad (7)$$
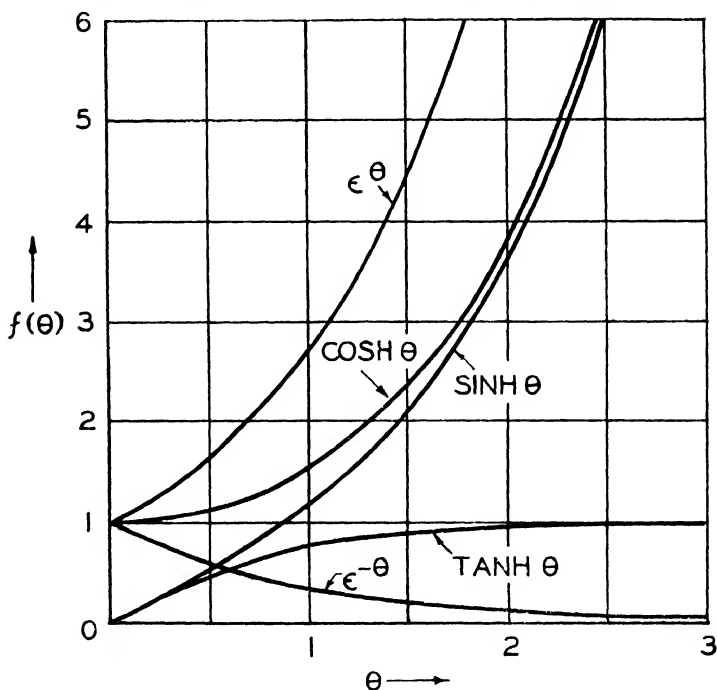
FIG. 18. CURVES OF HYPERBOLIC FUNCTIONS

Equation (7) gives the received current in terms of the applied e.m.f. and should be compared with that given for the leaky line for the d.c. case.

Infinite Line Conditions (Line Terminated in Characteristic Impedance Z_0). To determine the conditions of complete uniformity in the transmission it is assumed that the line is

continued without change to an infinite length. In this case pl is infinitely great.

Dividing the numerator and denominator in equation (6) by cosh pl gives—

$$I_s = \frac{V_s}{Z_0} \cdot \frac{Z_0 + Z_t \tanh pl}{Z_t + Z_0 \tanh pl}$$

When pl is infinitely large, tanh pl is equal to 1 and therefore

$$I_s = V_s/Z_0.$$

Under these conditions therefore the impedance presented by the line, that is the ratio V_s/I_s, is the characteristic impedance Z_0.

Referring to Fig. 18, from the graph of tanh θ it will be seen that for values of θ of 3 and upwards tanh θ is very nearly equal to 1. Thus so long as pl approaches or exceeds 3, the input impedance of the line is independent of the terminating conditions.

The infinite line conditions can be reproduced by making the terminating impedance equal to Z_0.

Substituting this value in equation (6) again gives the relation

$$I_s = V_s/Z_0.$$

Also from equations (3) by substituting this ratio we get

$$V_x = V_s \,(\cosh px - \sinh px)$$
$$I_x = I_s \,(\cosh px - \sinh px)$$
$$\therefore V_x/I_x = V_s/I_s = V_r/I_r = Z_0.$$

Under these conditions the ratio of volts to current is constant throughout the line and equal to Z_0.

Termination of the line by an impedance other than the characteristic impedance of the line leads to reflection of the electric wave and consequently increased power losses, and therefore it is usually arranged for a.c. telecommunication transmission lines to be terminated by an impedance as nearly equal to their characteristic impedance as practicable.

Any change of the ratio V/I brought about by change of impedance owing to discontinuity or change in the characteristics of the line leads to a reflection wave with consequent loss of power. Such changes are therefore avoided as far as possible.

Attenuation and Phase Constants. We have seen that when

the line is terminated in an impedance equal to the character-istic impedance, equations (3) may be written

$$V_x = V_s (\cosh px - \sinh px)$$
$$I_x = I_s (\cosh px - \sinh px)$$
$$\therefore \ V_x/V_s = I_x/I_s = \cosh px - \sinh px.$$

Substituting the exponential expressions for cosh and sinh gives

$$\frac{V_x}{V_s} = \frac{I_x}{I_s} = \frac{e^{px} + e^{-px}}{2} - \frac{e^{px} - e^{-px}}{2}$$

$$\therefore \ V_x/V_s = I_x/I_s = e^{-px}$$

Similarly

$$V_r/V_s = I_r/I_s = e^{-pl}$$

or

$$V_s/V_r = I_s/I_r = e^{pl}.$$

It will be seen that under these conditions a comparatively simple relationship exists between the sending voltage and current and the voltage and current at any point in the circuit. Furthermore, as these are the conditions under which lines are normally assumed to be worked, the foregoing simple relationship alone will be considered in what follows.

The propagation constant p is obtained from the formula

$$p = \sqrt{[(R + j\omega L)(G + j\omega C)]}$$

where R, L, G, and C are the primary constants of the line per unit length, p being the propagation constant per unit length; per mile of line in Great Britain.

The propagation constant is therefore a vector quantity and may be resolved into real and imaginary components thus—

$$p = a + jb$$

and

$$I_r/I_s = V_r/V_s = e^{-(al + jbl)} = e^{-al} e^{-jbl}$$

The factor e^{-al} determines the ratio, while the factor e^{-jbl} determines the phase of the sine wave, at the point considered, with respect to the sent wave.

Remembering that the voltages and currents in the whole of this treatment are sine waves we can write the expressions in full as follows—

$$I_r = e^{-al} I_s \sin(\omega t - bl)$$
$$\text{and } V_r = e^{-al} V_s \sin(\omega t - bl).$$

where bl is the angular phase difference in radians between the sent and received sine waves.

Graphs of the exponential functions e^{θ} and $e^{-\theta}$ (or $1/e^{\theta}$) are given in Fig. 18, and how the amplitude of the current or voltage wave is gradually reduced as the length of line increases will be clear from the curve for $e^{-\theta}$.

The significance of a change of phase between sine waves was illustrated in Fig. 15. The negative sign before the angle bl indicates that the phase of the received current lags behind that of the sent current as would be expected. As the angle is proportional to the length of line it is clear that the change of phase is progressive and linear, as distinct from the change of amplitude which is logarithmic. When the distance l between two points is such that $bl = 2\pi$, the sine waves at these points are at the same phase, as 2π corresponds to one complete cycle. Thus when the voltage or current is at a maximum at the first point it is also at a maximum at the second point. The distance l required to give this condition is called the *wavelength*, and is given by the relation

$$\text{Wavelength} = 2\pi/b.$$

For this reason b is sometimes called the *wavelength constant*. It is also called the *phase constant*.

a is called the *attenuation constant* as it determines the amount by which the signal wave is attenuated during transmission.

These constants can be obtained from the relation

$$p = a + jb = \sqrt{[(R + j\omega L)(G + j\omega C)]}.$$

Separating real and imaginary quantities gives

$$a = \sqrt{\tfrac{1}{2}\{\sqrt{[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]} + (GR - \omega^2 LC)\}} \quad (8)$$

$$b = \sqrt{\tfrac{1}{2}\{\sqrt{[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]} - (GR - \omega^2 LC)\}} \quad (9)$$

Useful approximate formulae for loaded cable conductors for which ωL is great compared to R, and G and is negligible compared to ωC, are—

$$a = (R/2)\sqrt{(C/L)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (10)$$

$$b = \omega\sqrt{(CL)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (11)$$

For unloaded cable conductors the inductance and leakance may be neglected at frequencies in the audio range, giving the approximate formulae.

$$a = b = \sqrt{(\tfrac{1}{2}\omega CR)} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (12)$$

The corresponding approximate formulae for the character-istic impedance are—

Loaded cable conductors

$$Z_0 = \sqrt{(L/C)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (13)$$

Unloaded cable conductors

$$Z_0 = \sqrt{(R/\omega C)}\backslash\overline{45}° \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (14)$$

The capital letters P, A, and B are used to denote the total propagation, attenuation, and phase characteristics of a trans-mission path and therefore for a uniform transmission line of length l miles.

$$P = pl, A = al, \text{ and } B = bl.$$

Distortionless Transmission. The occurrence of the ω terms in equation (8) indicates the dependency of the attenuation constant on the frequency of the applied voltage. Formula (12) shows this more clearly, and it is apparent that as the frequency increases so does the attenuation. It is obvious that complex waveforms or signals cannot be transmitted over such a line without distortion due to the reduction in amplitude of the higher frequency components with respect to the lower frequency components. Therefore a complex wave form cannot be transmitted faithfully unless the attenuation is the same for all the component frequencies.

Similarly equations (9) and (12) show that the phase constant is dependent on frequency. For distortionless transmission the various frequencies must appear at the receiving end with the same relative phases as they have in the initial complex wave-form. This condition is only fulfilled if the phase constant is proportional to frequency.

Heaviside saw that making the ratio $R/L = G/C$ would give the conditions necessary for distortionless transmission. Thus if we rewrite the formulae for the propagation constant and characteristic impedance as follows—

$$p = [\sqrt{(LC)}]\sqrt{\{[(R/L) + j\omega] [(G/C) + j\omega]\}}$$

$$Z_0 = \sqrt{\frac{L}{C}} \left\{ \sqrt{\left[\frac{(R/L) + j\omega}{(G/C) + j\omega}\right]} \right\}$$

it is easy to see that putting $R/L = G/C$ gives

$$p = [\sqrt{(LC)}] [(R/L) + j\omega] = [\sqrt{(C/L)}] (R + j\omega L).$$

and $Z_0 = \sqrt{(L/C)}$ $\qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (15)$

and is therefore independent of frequency.

Also
$$p = a + jb = R\sqrt{(C/L)} + j\omega\sqrt{(CL)}.$$

Whence the attenuation constant is given by

$$a = R\sqrt{(C/L)} = \sqrt{(RG)} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (16)$$

and is independent of frequency. The phase constant is given by

$$b = \omega\sqrt{(CL)} = 2\pi f\sqrt{(CL)} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (17)$$

and is therefore proportional to frequency.

The conditions for distortionless transmission are thus satisfied.

The primary constants for a pair in an underground paper core cable having 20 lb. conductors are approximately—

$$R = 88 \ \Omega/\text{mile.}$$
$$L = 1 \ \text{mH/mile.}$$
$$G = 1 \ \mu\text{mho/mile.}$$
$$C = 0\cdot062 \ \mu\text{F/mile.}$$

Thus
$$R/L = 88/10^{-3} = 88\ 000$$

and
$$G/C = 1/0\cdot062 = 16\cdot1.$$

It will be seen that for such a circuit the distortionless condition can only be approached with a very large increase in the inductance or leakance.

Increasing the leakance has the disadvantage of increasing the attenuation, and the method adopted is to increase the inductance.

Loading. Theoretically the minimum attenuation is secured when the distortionless condition is attained, but in practice inductance cannot be added to the circuit without affecting other constants. In a continuously loaded cable, in which the conductors are loaded by a wrapping of magnetic material, the resistance losses and capacitance are increased. In a coil-loaded cable the resistance losses are increased. Above a certain limit therefore the addition of inductance increases the attenuation.

Since the advent of the thermionic valve repeater, the securing of the minimum attenuation in the cable circuits has ceased to be of primary importance except in the case of long

submarine cables. The problem is now one of securing the correct balance between cost of cable and loading and repeater stations.

The coil loading of transmission circuits is complicated by the fact that a coil-loaded circuit acts as a low-pass filter and does not transmit any frequencies above a certain critical point called the *cut-off frequency*, and given approximately by the formula

$$f_c = 1/\pi\sqrt{(L_1 C_1)}$$

where L_1 is the inductance and C_1 the capacitance per loading section. Thus for a given cut-off frequency, if the inductance of the loading coils is increased, the distance between them must be correspondingly reduced so as not to vary the product $L_1 C_1$.

Alternatively, where it is required to transmit a wide band of frequencies such as on music and multi-channel carrier telephone circuits, the loading is reduced or even dispensed with altogether.

Fig. 19 gives the attenuation and phase characteristics for a typical underground cable circuit. Loaded circuits of this type are used for 18-channel voice-frequency telegraphy. It will be seen that when loaded the attenuation is almost constant over the range of frequencies 300 to 4 000 c/s, and the phase constant is almost proportional to frequency showing how, with the added inductance, the distortionless condition is approached.

TRANSMISSION UNITS

In practice it is necessary to have some means of assessing the transmission efficiency of circuits. The circuits are normally designed so that the attenuation is approximately constant over the band of frequencies to be transmitted, and phase distortion is negligible. If these conditions are fulfilled, the efficiency of the circuit can be measured by the volume of the received power for a given input power at some given representative frequency, usually 800 c/s for audio-frequency circuits. The transmission units in use are based on this assumption.

Neglecting phase difference, we have for a properly terminated line

$$I_r/I_s = V_r/V_s = e^{-al} = e^{-A} \qquad . \qquad . \qquad . \qquad . (18)$$

Also assuming non-reactive characteristic impedance the ratio of received to sent power is given by

$$W_r/W_s = I_r V_r/I_s V_s = e^{-2al} = e^{-2A} \qquad . \qquad . \qquad . \quad (19)$$

By way of example suppose a line has an attenuation constant of 0·11 per mile. The ratios for various lengths of line are
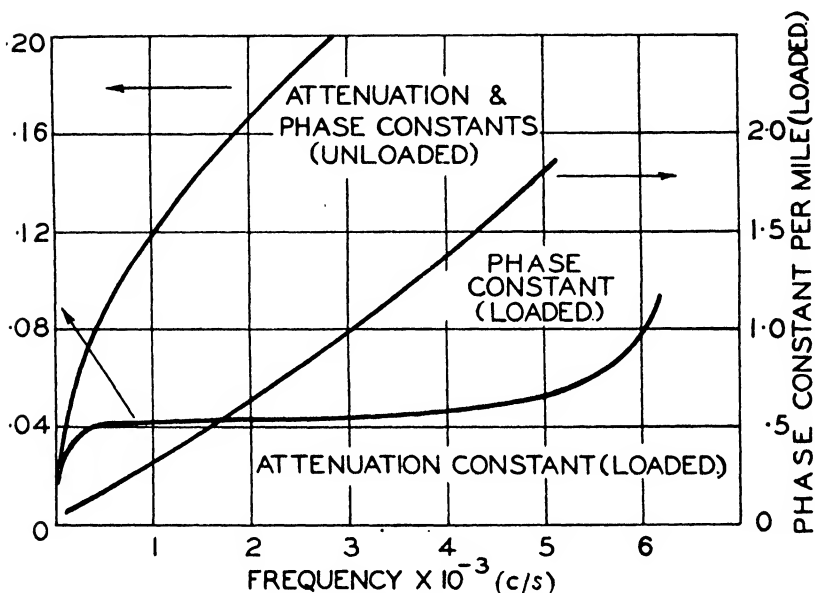
FIG. 19. ATTENUATION AND PHASE CHARACTERISTICS
FOR 25 LB CABLE PAIR CIRCUIT

(a) Unloaded
(b) Loaded with 44 mH loading coils at 2 000 yd spacing

given in the following table. (The value 0·11 has been chosen to show the point more clearly.)

Length of Line (miles)	al ($a = 0.11$)	$\dfrac{I_r}{I_s} = \dfrac{V_r}{V_s} = e^{-al}$	$\dfrac{W_r}{W_s} = e^{-2al}$
10	1·1	$\frac{1}{3}$	$\frac{1}{9}$
20	2·2	$\frac{1}{9}$	$\frac{1}{81}$
30	3·3	$\frac{1}{27}$	$\frac{1}{739}$
40	4·4	$\frac{1}{81}$	$\frac{1}{8561}$

Because $e^{-1\cdot1} = 0\cdot33$ (approximately), the received current at the end of ten miles is one-third of the sent current. The addition

of a second length of ten miles again reduces the current to one-third of that at the end of the first ten miles, giving an overall ratio of 1/9.

Thus if we wish to know the total transmission efficiency of two lengths of line, one having an attenuation value of 1·1 and the other 2·2, we can add these attenuation values, giving 3·3, but must multiply the current or power ratios.

$$I_r/I_s = \tfrac{1}{3} \times \tfrac{1}{9} = \tfrac{1}{27}$$

$$W_r/W_s = \tfrac{1}{9} \times \tfrac{1}{81} = \tfrac{1}{729}.$$

As it would be inconvenient to multiply the various ratios when determining the combined effect of various lines and apparatus, the transmission units are based on the attenuation values so that they can be added together.

The Decibel.   The unit of transmission used in this country and America is the *decibel* (db) formerly called a *transmission unit* (T.U.).   The fundamental unit is the *bel* (so named after Alexander Graham Bell) which can be defined as follows—

A transmission path for which the ratio of the power input to the received power is 10, *is said to have a transmission equivalent of one bel.*

Equation (19) can now be written.

$$W_r/W_s = e^{-2A} = 10^{-b}$$

or $\qquad W_s/W_r = e^{2A} = 10^b$

where b is the attenuation factor measured in bels.

It has been found convenient to make the practical unit one decibel which is, as its name implies, one-tenth of a bel, whence

$$W_s/W_r = 10^{db/10} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (20)$$

where db is the number of decibels.

Equation (20) may also be written

$$db = 10 \log_{10}(W_s/W_r).$$

It will be seen that by the choice of the unit for a ratio of 10 the base has been changed from e to 10. This simplifies the mental calculation of ratios for a given number of decibels and also enables ratios to be found by direct reference to ordinary logarithmic tables.

It is easy to see that for 20 db the ratio

$$W_s/W_r = 10^2 = 100,$$

For 30 db

$$W_s/W_r = 10^3 = 1\,000$$

and so on.

Assuming that the ratio

$$I_s/I_r = V_s/V_r$$

holds good, we have

$$I_s/I_r = V_s/V_r = \sqrt{(W_s/W_r)}.$$

Thus

$$I_s/I_r = V_s/V_r = \sqrt{(10^{\,db/10})} = 10^{\,db/20},$$

or $\qquad$ db $= 20 \log_{10} (I_s/I_r).$

Thus for 20 db, $I_s/I_r = 10$, and for 40 db, $I_s/I_r = 100$.

The attenuation factor, A for 1 db can be found as follows—

$$I_s/I_r = e^{\,A} = 10^{\,db/20}.$$

Now $\qquad \log_{10} e = 0{\cdot}4343$

$\qquad$ or $\qquad 10^{\,0{\cdot}4343} = e$

$\qquad \therefore \qquad 10^{\,0{\cdot}4343 A} = 10^{\,db/20}$

and $A = db/8{\cdot}686 = 0{\cdot}1151$ db;

or the attenuation factor A for a line having a transmission equivalent of 1db $= 0{\cdot}1151$.

The table of current and power ratios gives values from zero to 49 db. It will be seen that an increase of 10 db increases the power ratio by ten times, so that from the ratios for 1 to 10 db can be found the power ratio for any other number

TABLE OF CURRENT, VOLTAGE AND POWER RATIOS
FOR VARIOUS TRANSMISSION EQUIVALENTS

db	0	1	2	3	4	5	6	7	8	9
$\dfrac{I_s}{I_r}$ or $\dfrac{V_s}{V_r}$	1	1·122	1·259	1·413	1·585	1·778	1·995	2·239	2·512	2·818
$\dfrac{W_s}{W_r}$	1	1·259	1·585	1·995	2·512	3·162	3·981	5·012	6·310	7·943

db	10	11	12	13	14	15	16	17	18	19
$\dfrac{I_s}{I_r}$ or $\dfrac{V_s}{V_r}$	3·162	3·548	3·981	4·467	5·012	5·623	6·310	7·079	7·943	8·913
$\dfrac{W_s}{W_r}$	10	12·59	15·85	19·95	25·12	31·62	39·81	50·12	63·10	79·43

db	20	21	22	23	24	25	26	27	28	29
$\dfrac{I_s}{I_r}$ or $\dfrac{V_s}{V_r}$	10	11·22	12·59	14·13	15·85	17·78	19·95	22·39	25·12	28·18
$\dfrac{W_s}{W_r}$	100	125·9	158·5	199·5	251·2	316·2	398·1	501·2	631·0	794·3

db	30	31	32	33	34	35	36	37	38	39
or $\dfrac{V_s}{V_r}$	31·62	35·48	39·81	44·67	50·12	56·23	63·10	70·79	79·43	89·1
	1 000	1 259	1 585	1 995	2 512	3 162	3 981	5 012	6 310	7 943

db	40	41	42	43	44	45	46	47	48	49
or $\dfrac{V_s}{V_r}$	100	112·2	125·9	141·3	158·5	177·8	199·5	223·9	251·2	281·
	10 000	12 590	15 850	19 950	25 120	31 620	39 810	50 120	63 100	79 430

of db. For example, the power ratio for 1 db is 1·259, for 11 db, 12·59, and for 21 db, 125·9.

For the current ratios a difference of 20 db in the equivalent gives a difference of 10 times in the ratio, while a difference of 10 db gives a difference of 3·162 (i.e. $\sqrt{10}$) times.

The Neper. The attenuation unit used on the Continent is the *neper*, which is based on the current ratio instead of the power ratio and is obtained simply from the equation

$$I_s/I_r = e^N, \text{ or } N = \log e \, (I_s/I_r),$$

where N is the attenuation in nepers.

The ratio of sent to received current for an attenuation of 1 neper is therefore.

$$I_s/I_r = e = 2\cdot7183.$$

It will be seen that this unit has been obtained directly by replacing al by N in equation (18), and for this reason the neper is sometimes called the *natural attenuation unit*.

To convert decibels to nepers, or vice versa, we have—

$$1 \text{ neper} = 8\cdot686 \text{ decibels}$$

$$1 \text{ decibel} = 0\cdot1151 \text{ neper.}$$

Fig. 20 is a scale showing the relative magnitudes of the transmission units and the corresponding power, current and voltage ratios.

Use of Transmission Units for Expression of Repeater Gains and Transmission Levels. The gain of a repeater or amplifier may be expressed as a negative attenuation. Thus a repeater giving a gain in power ratio of 100 times would be said to give a gain of 20 db. This enables the overall equivalent of a repeatered trunk circuit to be calculated by deducting the total gains of all repeaters from the total losses.

The transmission level or volume of power at any point in a circuit when a given power is applied to one end is also quoted in decibels, the levels being quoted as − or + depending upon whether the power is smaller or greater than the input. A typical level diagram is shown in Fig. 21. It refers to a repeatered circuit having three repeater sections. The attenuations of the three sections are 9, 12, and 18 db, while the gains given by the three repeaters are 13, 13, and 7 db respectively. The overall transmission equivalent
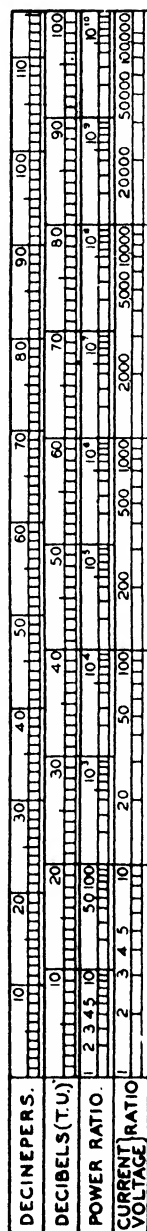

Fig. 20. Scale of Transmission Units and Corresponding Ratios

is therefore $39 - 33 = 6$ db. The corresponding level
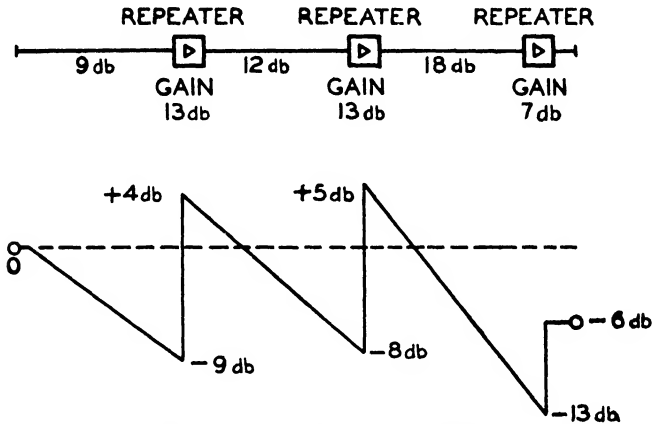diagram gives the power level at any point in the circuit

Fig. 21. Typical Level Diagram

for zero level at the sending end (taken as 1 mW for testing
purposes).

SECTION 4

RELÁTION BETWEEN STEADY-STATE AND TRANSIENT CONDITIONS

In the case of the distortionless line all frequencies are transmitted with equal attenuation and the same relative phase displacement. Thus any complex wave form applied at the sending end will be reproduced faithfully at the receiving end, as all the component frequencies will be transmitted with the correct relative amplitudes and phases.

In such a case the square-topped wave form shown in Fig. 16 would be transmitted without distortion. If we imagined that the speed of the reversals were extremely slow, they would appear to all practical purposes as isolated reversals of the battery at the sending end. The shape of the current curve at the receiving end would then correspond to the arrival curve. Thus for the distortionless line the arrival curve rises vertically to the steady value of the current, and there is no transient time.

A circuit which is distortionless according to a.c. steady-state theory—that is, for sinusoidal wave forms—is distortionless therefore for all wave forms, and there are no transient effects. It is clear also that the more a circuit departs from the distortionless condition the longer the transient conditions will persist; in other words the longer will be the time required for the received signals to build up to the steady state. It has been shown in the section on direct current transmission how telegraph distortion is related to the build-up time of the arrival curve. It is obvious therefore that there is a close relationship between the a.c. steady-state characteristics of the line (variation of attenuation and phase constants with frequency) and telegraph distortion. Devices such as shunted condensers and magnetic shunts are used on long d.c. telegraph lines to improve the shape of the received signals and reduce the telegraph distortion. In so far as these devices reduce the telegraph distortion by decreasing the effective build up time of the signals, they can be regarded as having achieved this by virtue of having improved the steady-state characteristics of the line, and this constitutes the readiest theoretical explanation whereby their action can be understood and the theory applied to practice.

Before pursuing the explanation of the action of distortion correction devices the connexion between steady-state and transient conditions will be further illustrated by reference to a method of obtaining the arrival curve direct from ordinary a.c. transmission theory.

Determination of Arrival Curve from the Steady-State Solution. The method about to be briefly explained and which has been dealt with fully by H. W. Malcom* was used by Breisig in 1900 to calculate the arrival curve of an Emden-Vigo cable.

It will be clear from what has been given earlier regarding the construction of telegraph signals that if square-topped reversals are sent into a circuit at a slow enough rate, the received current will to all intents and purposes build up to its final steady value between each reversal of the voltage, and the shape of the build-up curve will correspond to that of the arrival curve. Now considering the square-topped reversals as being composed of the harmonic series of sine waves as illustrated in Fig. 16 and given by equation (2), it will be apparent that the shape of the arrival curve can be obtained by calculating the amplitude and phase of the individual received sine waves of which the signals are composed. These calculations can be made by means of the ordinary steady-state formula which in its fullest form is—

$$I_r = \frac{E}{(Z_a + Z_t)\cosh pl + [Z_o + (Z_a Z_t / Z_o)]\sinh pl} \qquad . \quad (7)$$

This formula takes account of the terminal impedances at each end and all constants of the circuit.

It will be remembered that the Kelvin solution for the arrival curve neglects leakance and inductance and also assumes zero terminal impedances. This severely limits the practical use of the formula, because in the case of all lines where the shape of the arrival curve becomes of importance, special terminal networks are used to improve the signal shape, and thus the arrival curve obtained in practice would bear little resemblance to the Kelvin curve.

The equation for periodic reversals of the battery may be written

$$Ef(t) = \frac{4E}{\pi} \sum_{m=0}^{m=\infty} \frac{1}{2m+1} \sin(2m+1)\omega t . \qquad . \qquad . \quad (21)$$

* H. W. Malcolm, *The Theory of the Submarine Telegraph and Telephone Cable:* Benn, London, 1917.

where the sign Σ indicates the summation of the terms with all values $m = 0, 1, 2, 3$, etc., to infinity.

Substituting in equation (7) gives

$$I_r = \frac{4E}{\pi} \sum_{m=0}^{m=\infty} \frac{\dfrac{1}{2m+1} \sin (2m+1)\,\omega t}{(Z_a + Z_t) \cosh pl + \left(Z_o + \dfrac{Z_a Z_t}{Z_o} \right) \sinh pl} \quad (22)$$

where $\omega = 2\pi/T = 2\pi f$; and

$T =$ the period in seconds of one complete cycle.

The values of Z_a, Z_t and p must be calculated for each term in the series, the values of ω used being ω, 3ω, 5ω, etc.

Equation (22) is the complete solution for the response of a circuit to square-topped reversals, and if T is taken sufficiently large for the current to be sensibly equal to the final steady-state value in the duration $T/2$ sec of one half-cycle, it becomes a solution for the arrival curve. As such it is of more importance from the facts that can be learnt from an examination of the factors contained in it than from its use for the actual calculation of the arrival curve. Save in certain exceptional cases the terms of the series converge but slowly, and the labour in the arithmetical computation is considerable.

The first point of interest which emerges from equation (22) is that it is symmetrical in Z_a and Z_t, which shows that the effect of a given terminal impedance on the arrival curve is independent of whether it is at the sending or receiving end. This should be compared with the d.c. solution for a line with resistance at the sending and receiving ends which shows the same effect of symmetry.

For most cases in practice, and in all cases where the shape of the arrival curve is of primary importance, the line is so long that pl is great enough for sinh pl to be taken as equal to cosh pl, and equation (22) simplifies to—

$$I_r \doteqdot \frac{4E}{\pi} \sum_{m=0}^{m=\infty} \frac{\dfrac{1}{2m+1} \sin (2m+1)\,\omega t}{\tfrac{1}{2}\left(Z_a + Z_t + Z_o + \dfrac{Z_a Z_t}{Z_o} \right) e^{pl}} \quad . \quad . \quad (23)$$

or if Z_a, as is very frequently the case, is small compared to Z_o and Z_t we have

$$I_r \doteqdot \frac{4E}{\pi} \sum_{m=0}^{m=\infty} \frac{\dfrac{1}{2m+1} \sin (2m+1)\,\omega t}{\tfrac{1}{2}(Z_o + Z_t)\, e^{pl}} \quad . \quad . \quad . \quad (24)$$

It is obvious that it is the denominator of the term under the summation sign in equations (22), (23), and (24) which controls the mode of build-up of the arrival curve. It will be seen that apart from varying the constants of the line itself, the only method of improving the received signals is by manipulation of the terms Z_a and Z_t, that is, the sending and receiving terminal impedances. This will be illustrated by reference to an actual case.
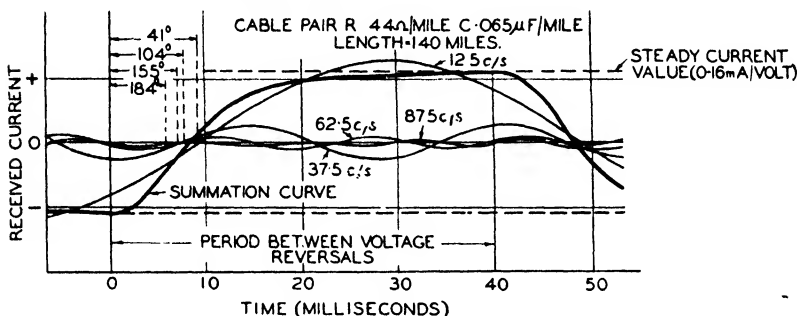

FIG. 22. ARRIVAL CURVE CONSTRUCTED FROM SINE-WAVE COMPONENTS
OF SQUARE-TOPPED REVERSALS
(Zero Terminal Impedances)

Calculation of Arrival Curve with Zero Terminal Impedances
(Kelvin Curve). Fig. 22 gives the arrival curve for an underground cable pair having the following constants—

$$R = 44 \ \Omega/\text{mile}.$$

$$C = 0 \cdot 065 \ \mu\text{F}/\text{mile}.$$

$$l = 140 \ \text{miles}.$$

(Leakance and inductance negligible.)

Such an arrival curve can, of course, be calculated readily from the Kelvin equation given in the preceding section, but to illustrate the use of terminal correcting networks it is preferable to use the steady-state method.

We have $CRl^2 = 56 \times 10^{-3}$

$\qquad\qquad\quad = 56 \ \text{msec}.$

and $0 \cdot 55 CRl^2 = 31 \ \text{msec}.$

When $t = 0 \cdot 55 CRl^2$, the current will have almost reached the steady value. This relation between t and CRl^2 was given in the section on direct current transmission.

To calculate the arrival curve let it be assumed therefore that square-topped reversals are impressed at the sending end of the line, the time interval between reversals being 40 msec. This gives the periodicity of the reversals as $1\,000/80 = 12\cdot5$ c/s, and the current will build up to the steady value within the half-period of 40 msec.

From equation (24) we have

$$I_r = \frac{4E}{\pi} \sum_{m=0}^{m=\infty} \frac{\frac{1}{2m+1} \sin(2m+1)\,\omega t.}{Z_o\left(\frac{1}{2}e^{al}\,e^{jbl}\right)}$$

For $m = 0$, $f = 12\cdot5$ c/s and $\omega = 78\cdot5$.

Z_o, a and b can be calculated from the approximate formulae

$$Z_o = \sqrt{(R/\omega C)}\ \overline{\backslash 45^\circ} = 2\,940\ \overline{\backslash 45^\circ}$$
$$al = bl = l\sqrt{(\tfrac{1}{2}\omega CR)} = 1\cdot5.$$

bl gives the angular phase displacement of the sine wave in radians.

We have therefore for the fundamental frequency of the reversals

$$I_r = \frac{1\cdot27\,E \sin 78\cdot5\,t}{2\,940\ \backslash 45^\circ \times 2\cdot35\ /86^\circ}$$
$$= \frac{1\cdot27\,E \sin 78\cdot5\,t}{6\,900\ /41^\circ}$$

Similarly for the third, fifth, and seventh harmonics we have—

$$I_r = \tfrac{1}{3}\,\frac{1\cdot27\,E \sin 3\omega t}{1\,700\ \backslash 45^\circ \times 6\cdot73\ /149^\circ}$$

$$I_r = \tfrac{1}{5}\,\frac{1\cdot27\,E \sin 5\,\omega t}{1\,300\ \backslash 45^\circ \times 16\cdot6\ /200^\circ}$$

$$I_r = \tfrac{1}{7}\,\frac{1\cdot27E \sin 7\omega t}{1\,100\ \backslash 45^\circ \times 27\cdot3\ /229^\circ}$$

The value of the steady current is given by $E/6\,150$.

If we take the value of the steady current as unity, the amplitude and phase of each frequency is as follows—

$12\cdot5$ c/s $\qquad I_r = 1\cdot21\ \backslash\overline{41^\circ}\ \sin \omega t.$

$37\cdot5\quad,,\qquad I_r = 0\cdot24\ \backslash\overline{104^\circ}\ \sin 3\omega t.$

$62\cdot5\quad,,\qquad I_r = 0\cdot08\ \backslash\overline{155^\circ}\ \sin 5\omega t.$

$87\cdot4\quad,,\qquad I_r = 0\cdot04\ \backslash\overline{184^\circ}\ \sin 7\omega t.$

The correct relative amplitudes of the harmonics with respect to the fundamental as they would appear in the square-topped reversals would be 0·4, 0·24, and 0·17 respectively.

These sine waves are plotted in Fig. 22.

It will be observed how the increasing attenuation at the higher frequencies causes the harmonics to be reduced to an increasing degree as the frequency increases. Also, because the phase constant b is not proportional to frequency, the higher frequencies have a smaller phase displacement and are therefore out of phase with the fundamental.
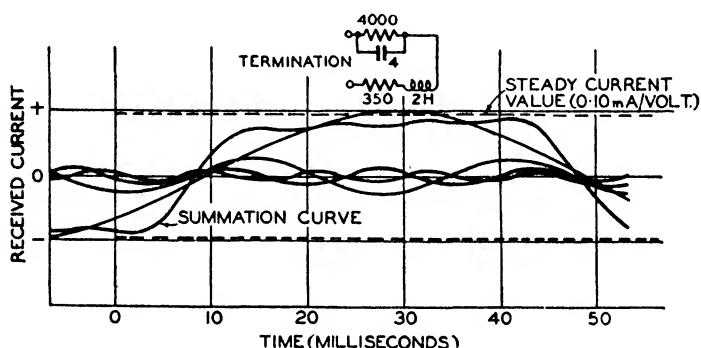
Fig. 23. Arrival Curve Constructed from Sine-wave Components of Square-topped Reversals
(Shunted Condenser and Relay Termination)

The combined effect of this distortion is to slow up the build-up of the received wave, represented by the summation curve. It will be observed that the seventh harmonic is very much reduced and has little effect on the received current.

Fig. 22 shows clearly the effect of attenuation distortion and phase distortion on the arrival curve.

DISTORTION CORRECTING NETWORKS

Shunted Condenser. Fig. 23 has been drawn for the same line, terminated by a shunted condenser and telegraph relay having the following constants—

Condenser .	.	.	.	. 4 μF.
Shunting resistance	.	.	. 4 000 Ω.
Inductance of relay	.	.	. 2 H.
Resistance of relay	.	.	. 350 Ω.

The formula in this case is equation (24).

The a.c. impedance of a shunted condenser is given by—

$$\frac{R\,(1 - j\omega CR)}{1 + \omega^2 C^2 R^2} = \frac{R}{\sqrt{(1 + \omega^2 C^2 R^2)}} \underline{/\tan^{-1} - \omega CR}.$$

and the impedance of the relay, by

$$R_x + j\omega L.$$

The separate and combined impedances of these two elements at the requisite frequencies are given below—

Frequency (c/s)	Impedance (approx.) (Ω)		
	Shunted Condenser	Relay	Combined
12·5	1 540 − j1 930	350 + j160	1 890 − j1 770
37·5	260 − j990	350 + j480	610 − j510
62·5	105 − j640	350 + j800	455 + j160
87·5	50 − j440	350 + j1 100	400 + j660

Due therefore to the presence of the shunted condenser the value of the terminal impedance falls with rise of frequency, and the value of the term $(Z_o + Z_t)$ of equation (24) at the various frequencies is as follows—

Frequency (c/s)	$Z_o + Z_t$	
12·5	5 500	\45°
37·5	2 480	\43°
62·5	1 580	\29°
87·5	1 200	\6°

The added terminal resistance reduces the steady current value to

$$E/10\ 500.$$

Again taking this value as unity gives—

Frequency (c/s)	I_r	
12·5	1·03	\41°
37·5	0·26	\106°
62·5	0·10	\171°
87·5	0·06	\223°

The correct relative amplitudes of the harmonics would in this case be 0·34, 0·21, and 0·15 respectively. The improvement obtained by use of the shunted condenser is therefore apparent. Some improvement in phase displacement is also secured, the angles being more nearly proportional to frequency. It will be seen that although the shunted condenser reduces the distortion, there is some reduction in amplitude of the signals, the amplitude in the first case being 0·16 mA/V whereas in the second case it is reduced to 0·10 mA/V.

The sine waves are plotted in Fig. 23 and the improvement in the arrival curve will be seen. The effective build-up time has been reduced by about 30 per cent. In making this comparison it should also be remembered that the curve in Fig. 22 refers to the case when the receiving end was short-circuited.

It will be clear from equation (22) and the remarks relating to it that distortion can equally be counteracted by the use of a shunted condenser at the sending end.

This use of the shunted condenser will be recognized as that of bridge duplex with resistance arms shunted by a condenser.

In submarine cable practice the resistance may be omitted altogether to secure the maximum attenuation at low frequencies.

Inductive Shunt (or Magnetic Shunt). Similar effects to those of the shunted condenser may be secured by shunting the relay by a resistance and inductance in series. In this case, instead of connecting an impedance which falls with frequency in series, an impedance which rises with frequency is connected in parallel. This shunt path absorbs a considerable proportion of the low-frequency components, but at higher frequencies a large proportion of the received current passes through the relay.

Similarly to the shunted condenser the inductive shunt may be used at the sending end. In order that the shunting effect shall be produced on the current sent to line, it is necessary for a suitable impedance to be included in the battery circuit; otherwise the inductive shunt would have practically no effect, merely imposing a heavy load on the battery.

The sending end inductive shunt is a convenient method of distortion correction on differential duplex circuits. Where this method has been used in the British Post Office, on account of the heavy drain imposed on the source by the shunt, rectifier units have been used and the power taken direct from the supply mains.

Adjustment of Distortion Correcting Networks. The best value of components to be used in any given case depends on the speed at which the circuit is to. be worked. In the case of long circuits, proper correction over a wide range of frequencies so as to cover various working speeds would necessitate the use of complicated networks.

It is of course the shorter signals which suffer the most distortion; for example, in the teleprinter code an isolated 20 msec signal would be most seriously affected, and the network would be adjusted particularly to take care of such signals. If a vibrating relay circuit were in use, this would look after the 20 msec signals, and any distortion correcting network would most probably be adjusted with more reference to 30 msec (stop) and 40 msec signals. Care has to be taken, however, in setting the values of the components in such cases of selective distortion correction on long lines, because the network may not produce true distortion correction but rather introduce a form of anti-distortion by increasing the amplitude of certain selected frequency components of the signals. It is thus possible to adjust the network so that signal elements of a certain length are received with practically no distortion, while other elements may be adversely affected, and in general the values of components are a compromise obtained by making tests on all representative signal combinations, including long and short signal elements.

It has been found that by far the best results are secured for the purposes of land telegraphy by adjusting the networks while observing the received signals on a telegraph distortion-measuring set, the best all-round value being chosen from observations on various signal combinations.

SECTION 5

ELECTRICAL WAVE FILTERS

AN electrical wave or frequency filter consists of an arrangement of electrical components, generally inductors and condensers, having two input and two output terminals, through which currents of certain selected frequencies can pass with very little loss of energy, while all other frequencies are attenuated to such an extent as to be of negligible account in the output. Filters are used extensively in telecommunications; in particular multi-channel telephone and telegraph systems depend for their success on the use of efficient filters.

The various types of filters are low-pass, high-pass, band-pass, and band-stop.

A *low-pass filter* will pass freely all frequencies from zero up to the cut-off frequency.

A *high-pass filter* will pass freely frequencies above the cut-off frequency, frequencies between zero and the cut-off frequency being attenuated.

A *band-pass filter* only passes frequencies in a band between two frequencies, the upper and lower cut-off frequencies of the filter. A band-pass filter is in effect a combination of low- and high-pass filter, the cut-off of the low-pass overlapping that of the high-pass.

A *band-stop filter* passes freely all frequencies outside a band between the upper and lower cut-off frequencies.

The attenuation characteristics of simple basic filter sections are given in Fig. 24. In practice the rate of increase of attenuation at the cut-off frequency can be improved by making use of modified forms of the basic sections. As the attenuation is proportional to the number of sections, improved discrimination against the unwanted frequencies can be obtained using filters composed of several sections.

Filters only behave properly if they are worked between impedances approximately equal to their own input and output impedance.

Elements of Filter Theory. The simplest form of filter is the so-called *tuned circuit* consisting of an inductance and capacitance in series or parallel. Such an arrangement can be used as a band-pass filter, as in a radio receiver, or as a bandstop

filter to eliminate certain unwanted frequencies from a signal wave.

Series Tuned Circuit. The impedance of the series tuned circuit shown in Fig. 25 is given by

$$Z \underline{/\theta} = R + j\,[\omega L - (1/\omega C)]$$

$$= \sqrt{\{R^2 + [\omega L - (1/\omega C)^2]\}}\, \underline{/\tan^{-1}\dfrac{[\omega L - (1/\omega C)]}{R}}.$$

R is the resistance of the inductor and should be as small as possible for most efficient results.

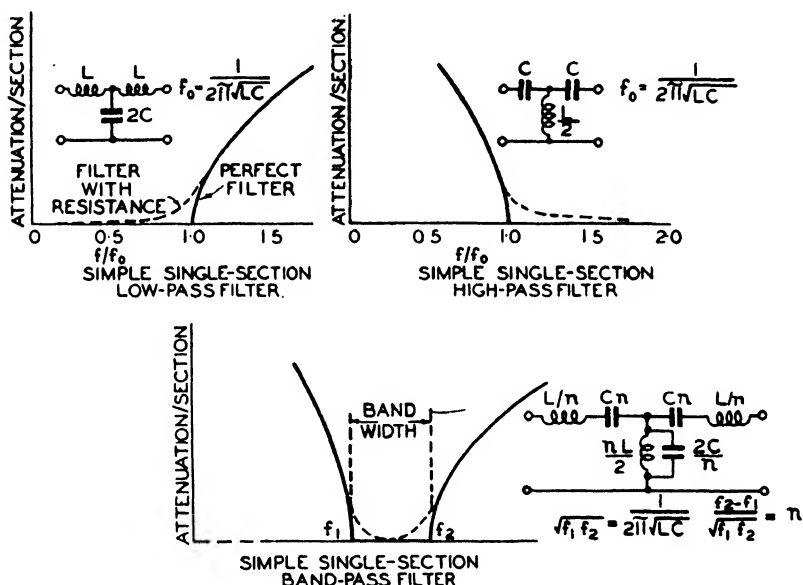FIG. 24. ATTENUATION CHARACTERISTICS OF SIMPLE FILTER SECTIONS

The condition of resonance is given when

$$\omega L = 1/\omega C,$$

and $\omega = \sqrt{(1/LC)}$ or $f = (1/2\pi)\sqrt{(1/LC)}$

When the frequency is such as to satisfy this equation the impedance is non-reactive and equal to R, and if R is very small the circuit is virtually short-circuited.

A graph of the modulus of the impedance of a series resonant

circuit is given in Fig. 25. The following values have been taken—

$L = 0 \cdot 1$ H ;
$C = 0 \cdot 4 \times 10^{-6}$ F $(0 \cdot 4\ \mu\text{F})$;
$R = 10\ \Omega$.

The resonant frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{4 \times 10^{-8}}} = 796 \text{ c/s.}$$

Parallel Tuned Circuit. The same components arranged as a parallel resonance circuit are shown in Fig. 26 together with the relative impedance frequency characteristic.

The impedance in this case is given by

$$Z\ \underline{/\theta} = \frac{\sqrt{\{R^2 + \omega^2[L(1 - \omega^2 LC) - CR^2]^2\}}}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

$$\underline{/\ \frac{\tan^{-1}\omega[L(1 - \omega^2 LC) - CR^2]}{R}}$$

When $\omega L = 1/\omega C$.

$$Z\ \underline{/\theta} = \frac{R\sqrt{(1 + \omega^2 C^2 R^2)}}{\omega^2 C^2 R^2.}\ \underline{/\tan^{-1}\omega CR}$$

In general, $\omega^2 C^2 R^2$ is very small compared to unity; for example, with the present components it is 4×10^{-4}, and the impedance at resonance is very nearly

$$Z = 1/\omega^2 C^2 R^2 = \omega^2 L^2/R,$$
for $L = 0 \cdot 1$ H ;
       $C = 0 \cdot 4 \times 10^{-6}$ F ;
       $R = 10\ \Omega$.

The impedance at the resonant frequency is therefore 25 000 Ω.

When R is very small, terms containing R may be neglected except at frequencies close to resonance, which gives the impedance as

$$Z = \omega L/(1 - \omega^2 LC)$$

except close to resonance.

If the inductance were perfect and R therefore zero, the impedance would be infinite at resonance. At all frequencies
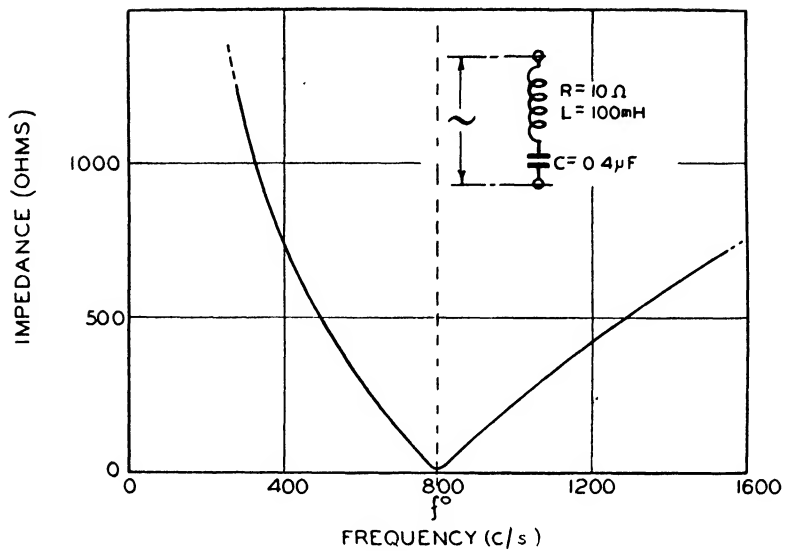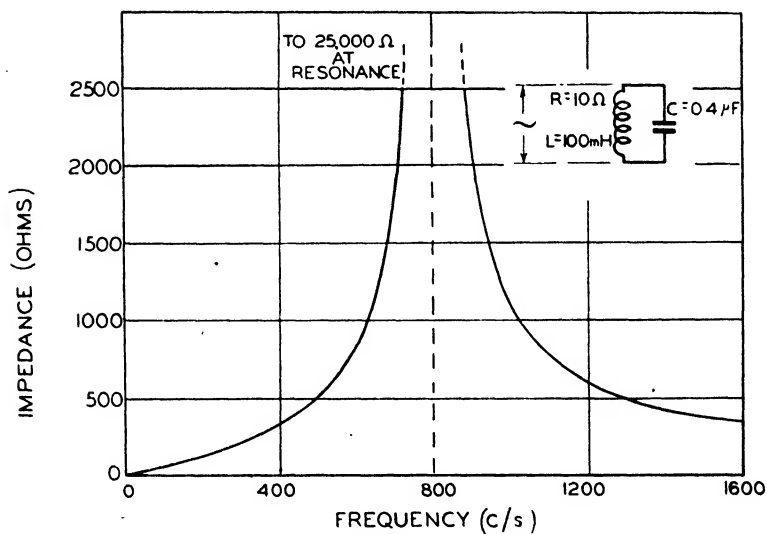
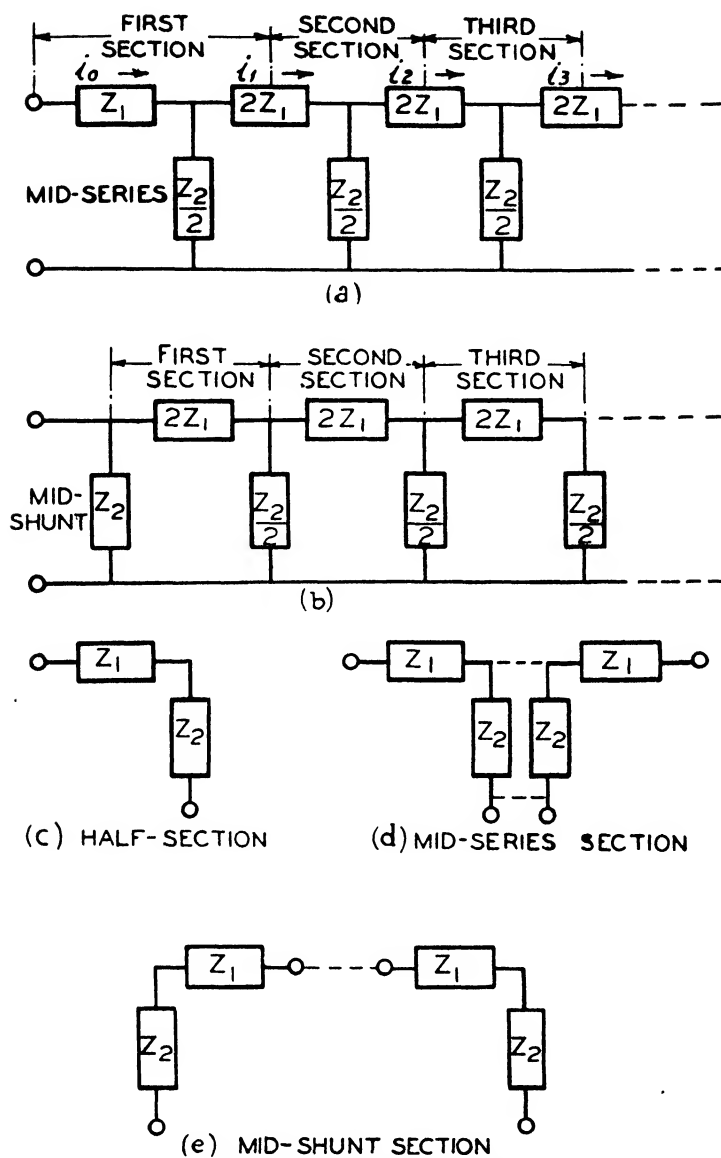FIG. 25. SERIES RESONANT CIRCUIT

FIG. 26. PARALLEL RESONANT CIRCUIT

below resonance the network would behave as an inductance and at frequencies above as a capacitance.

In both the foregoing circuits the effects at resonance are due to the coincidence of the supply frequency with the natural oscillation frequency of the tuned circuit.

Recurrent Networks. The theory of electric wave filters is best developed by reference to the theory of recurrent networks.

Fig. 27 shows the structure of recurrent ladder networks, (a) being a network with *mid-series* termination, and (b) with *mid-shunt* termination. These networks can both be made up from the same "half-section" shown at (c), having a series arm impedance Z_1 and shunt arm impedance Z_2. The networks can be considered as made up of half-sections back to back in pairs commencing as in (d) or (e). In either case the resultant shunt impedance in the ladder network is $Z_2/2$.

By considering these networks to be continued indefinitely, expressions may be deduced for their characteristic impedance and propagation constant.*

For the mid-series termination, shown in Fig. 27 (a), the characteristic impedance is given by—

$$Z_{01} = \sqrt{(Z_1{}^2 + Z_1 Z_2)} \quad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (25)$$

and for the mid-shunt termination, Fig. 27 (b) by

$$Z_{02} = Z_1 Z_2 / \sqrt{(Z_1{}^2 + Z_1 Z_2)} \quad . \qquad . \qquad . \qquad . \qquad . \quad (26)$$

As in the case of ordinary transmission lines, infinite line conditions can be obtained by terminating the network by the proper impedance. In practice, therefore, a filter only behaves properly if its characteristic impedance is approximately equal to that of the circuit in which it is connected.

Balanced and Unbalanced Networks. The networks shown in Fig. 27 are of the unbalanced type. The balanced type are shown in Fig. 28. The balanced type are obtained by dividing the series impedance of the corresponding unbalanced type equally between the two sides of the network.

It will be seen that the component impedances have all been related to the impedances Z_1 and Z_2 of the unbalanced half-section. This method gives uniformity in the formulae for the impedance and cut-off frequencies in all types of filters and has been followed throughout.

* See Appendix for proof of formula.

FIG. 27. LADDER NETWORKS
(Unbalanced Types)

Filters result when the impedances Z_1 and Z_2 contain re-active components. If the inductances and condensers used were perfect, that is to say, the impedances Z_1 and Z_2 were wholly re-active, the filters would pass frequencies in the pass range without any attenuation whatsoever, while at the cut-off

(a.) HALF-SECTION   (b) MID-SERIES SECTION
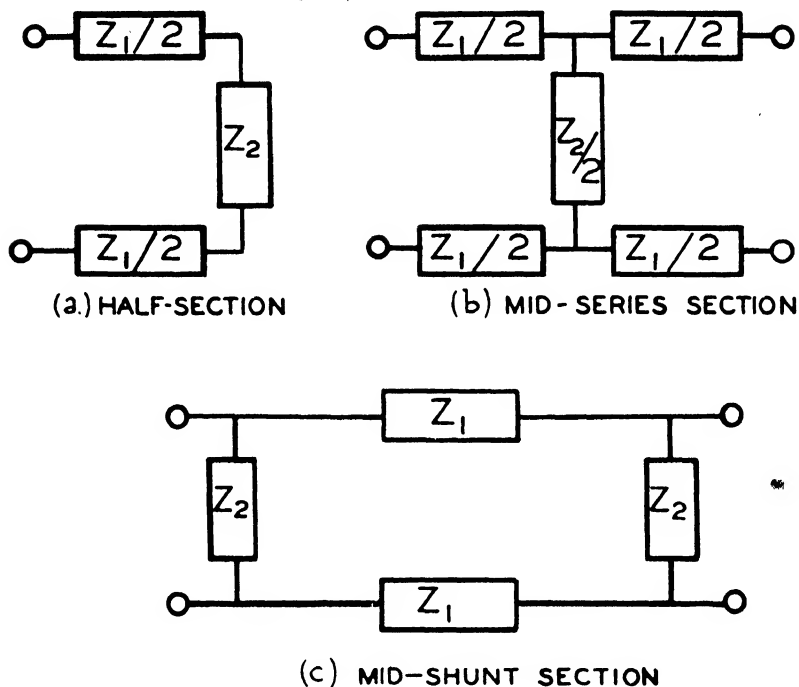

(c) MID—SHUNT SECTION

FIG. 28. LADDER NETWORKS
(Balanced Types)

frequency the attenuation would rise suddenly and steeply as shown in Fig. 24.

In practice, although the losses in the condensers may be negligible, some resistance loss is inevitable in the inductances and a small amount of attenuation is present in the pass range, and the attenuation commences to rise, as shown by the dotted curves in Fig. 24 before the cut-off frequency is reached. Resistance losses will, however, be neglected in the following treatment.

Low-Pass Filters. If the series impedance Z_1 of the ladder

network consists of a perfect inductance, and the shunt impedance Z_2 a condenser, we have—

$$Z_1 = j\omega L$$
$$Z_2 = 1/j\omega C.$$
$$\therefore Z_{0T} = \sqrt{[(L/C) - \omega^2 L^2]}$$
$$= \sqrt{(L/C)}\sqrt{(1 - \omega^2 LC)}. \qquad . \qquad . \qquad . \qquad (27)$$

and
$$Z_{02} = \frac{\sqrt{(L/C)}}{\sqrt{(1 - \omega^2 LC)}} \qquad . \qquad . \qquad . \qquad . \qquad (28)$$

In each of the foregoing the negative sign results because of the product of the two imaginary terms j, (i.e. $\sqrt{-1}$).

It will be seen that when ω has such a value that

$$\omega^2 LC = 1$$

the mid-series impedance is zero and the mid-shunt infinity.

Calling this value ω_0 we have

$$\omega_0 = 1/\sqrt{(LC)} = 2\pi f_0$$
$$\text{or } f_0 = 1/2\pi\sqrt{(LC)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (29)$$

f_0 is the cut-off frequency.

For values of ω less than ω_0, $\omega^2 LC$ is less than unity, and the impedances Z_{01} and Z_{02} are real or non-reactive. When $\omega^2 LC$ is greater than unity the expression under the root sign becomes negative and the factor j $(\sqrt{-1})$ re-appears, showing that the characteristic impedance is now wholly re-active and the network ceases to transmit power at the applied frequency.*

When the value of ω is small compared to ω_0, $\omega^2 LC$ is small compared to unity and the characteristic impedance is given approximately by

$$R_0 = \sqrt{(L/C)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (30)$$

The value of R_0 is equivalent to a resistance, and in designing a filter the values L and C are chosen so that R_0 is equal to the impedance of the circuit in which the filter is to work.

From the two formulae

$$f_0 = 1/2\pi\sqrt{(LC)}$$
$$\text{and} \qquad R_0 = \sqrt{(L/C)}$$

the values of L and C required in any given case can be calculated.

* See Appendix for a formal explanation of the behaviour of filters based on the effect of frequency on the propagation constant.

We have

$$L = R_0/2\pi f_0 \text{; and}$$
$$C = 1/2\pi f_0 R_0.$$

By way of example, the values of L and C for a low-pass filter having a cut-off frequency of 300 c/s and impedance of 600 Ω are as follows—

$$L = \frac{600}{6\cdot 28 \times 300} = 0\cdot 318 \text{ H.}$$

$$C = \frac{1}{6\cdot 28 \times 300 \times 600} = 0\cdot 885 \times 10^{-6}$$
$$= 0\cdot 885 \ \mu\text{F.}$$

L = 0·318 H   **0·318 H**

2C = 1·770 μF

(a.) LOW - PASS

C = 0·885 μF   **0·885 μF**

$$\frac{L}{2} = 0\cdot 159 \text{ H}$$
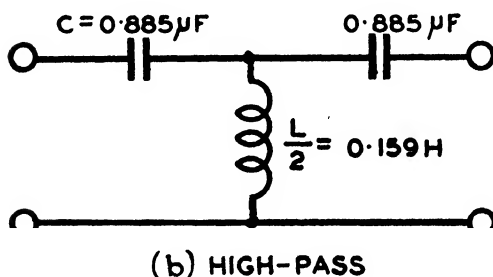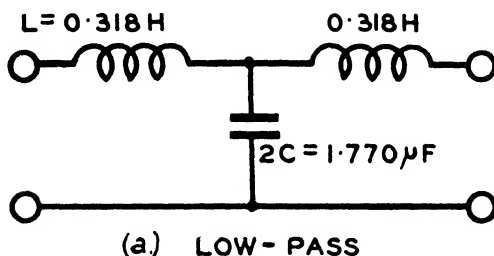
(b) HIGH–PASS

FIG. 29. SIMPLE MID-SERIES FILTER SECTIONS
Cut-off frequency (f_0) = 300 c/s
Impedance (R_0) = 600 Ω

It should be remembered that the values L and C are those for a half-section. For a mid-series full section such as shown in Fig. 27 the condenser would be 1·770 μF, that is, two shunt impedances in parallel. Such a filter is shown in Fig. 29.

The characteristic impedance of a basic type low-pass filter

is shown in Fig. 31. This is the impedance that would be measured if a large number of sections were connected in series to give "infinite line" conditions.

High-Pass Filters. In this case the series impedance Z_1 consists of a condenser, while the impedance Z_2 consists of an inductance and we have

$$Z_1 = 1/j\omega C;$$
$$Z_2 = j\omega L;$$

and

$$Z_{01} = \sqrt{\left(\frac{L}{C} - \frac{1}{\omega^2 C^2}\right)}$$
$$= \left(\sqrt{\frac{L}{C}}\right)\sqrt{\left(1 - \frac{1}{\omega^2 LC}\right)} \quad . \quad . \quad . \quad . \quad (31)$$

$$Z_{02} = \frac{\sqrt{(L/C)}}{\sqrt{[1 - (1/\omega^2 LC)]}} \quad . \quad . \quad . \quad . \quad (32)$$

Again, when $\omega^2 LC = 1$ the mid-series impedance is zero and the mid-shunt infinite, and we have

$$\omega_0 = 1/\sqrt{(LC)},$$
$$\text{and } f_0 = 1/2\pi\sqrt{(LC)} \quad . \quad . \quad . \quad . \quad . \quad (33)$$
$$\text{also } R_0 = \sqrt{(L/C)}. \quad . \quad . \quad . \quad . \quad . \quad (34)$$

In this case, however, for values of ω greater than ω_0—that is, for frequencies higher than the cut-off frequency (f_0)—$1/\omega^2 LC$ is less than unity, and the expression under the root sign is positive and the impedance is resistive. For frequencies less than f_0 the term $1/\omega^2 LC$ is greater than unity, and the impedance is therefore re-active at frequencies below the cut-off frequencies.

The filter therefore only transmits frequencies above the cut-off.

The values of L and C are given as before by—

$$L = R_0/2\pi f_0;$$
$$C = 1/2\pi f_0 R_0.$$

For a high-pass filter with a cut-off frequency of 300 c/s and impedance of 600 Ω we have again

$$L = 0.318 \text{ H};$$
$$C = 0.885 \ \mu\text{F}.$$

In this case, however, for a mid-series section the condensers used are equal to C, whereas the inductance is equal to $L/2$, that is 0.159 H. Such a filter is shown in Fig. 29.

The characteristic impedance is shown in Fig. 31.

Band-Pass Filters. Band-pass filters may be made up in a variety of ways. It is obvious that a band-pass filter can be made up of a low-pass filter and high-pass filter in series, the cut-off frequency of the low-pass filter being above that of the high-pass filter, the pass band being that lying between the two cut-off frequencies. Where a wide band of frequencies is to be passed, this method would usually be adopted.

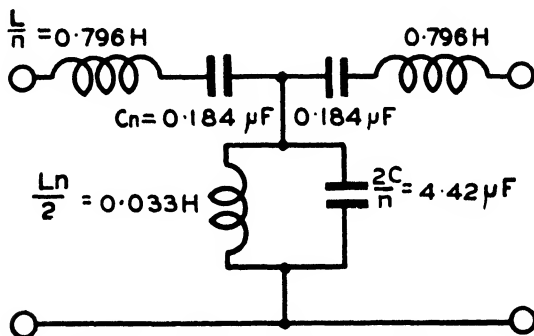The basic type of band-pass filter is shown in Fig. 30 in

FIG. 30. BASIC TYPE BAND-PASS FILTER

Mid-band frequency = 420 c/s
Frequency band = 120 c/s
Impedence (R_0) = 600 Ω

which the series impedance Z_1 is a series resonant circuit, and the shunt impedance Z_2 is a parallel resonant circuit. This arrangement of elements acts as a band-pass filter so long as the product of the series inductance and capacitance is equal to the product of the shunt inductance and capacitance.

We have therefore

$$Z_1 = \frac{j\omega L}{n} - \frac{j}{\omega C n};$$

$$Z_2 = \frac{-jn\,(L/C)}{\omega L - (1/\omega C)}.$$

Therefore

$$Z_{01} = \sqrt{\left(\frac{L}{C}\right)}\sqrt{\left(1 - \frac{(1 - \omega^2 LC)^2}{\omega^2 LC n^2}\right)} \qquad . \qquad . \qquad . \quad (35)$$

$$Z_{02} = \frac{\sqrt{\dfrac{L}{C}}}{\sqrt{\left(1 - \dfrac{(1 - \omega^2 LC)^2}{\omega^2 LC n^2}\right)}}. \qquad . \qquad . \qquad . \quad (36)$$

The cut-off frequencies are in this case given by the formulae

$$\sqrt{(f_1 f_2)} = 1/2\pi\sqrt{(LC)}; \quad (f_2 - f_1)/\sqrt{f_1 f_2} = n,$$

f_2 being the higher and f_1 the lower cut-off frequency.

Also we have

$$R_0 = \sqrt{(L/C)}.$$

As before the values of the inductances and capacitances can be calculated from the required cut-off frequencies and the impedance which is desired for R_0.

Thus

$$L = R_0/2\pi\sqrt{(f_1 f_2)};$$
$$C = 1/2\pi R_0\sqrt{(f_1 f_2)}.$$
$$\therefore\ L/n = R_0/2\pi(f_2 - f_1); \quad Cn = (f_2 - f_1)/2\pi f_1 f_2 R_0.$$
$$Ln/2 = R_0(f_2 - f_1)/4\pi f_1 f_2; \quad 2C/n = 1/\pi R_0(f_2 - f_1).$$

It will be observed that the value L/n of the series inductance and $2C/n$ of the shunt capacitance is only dependent on R_0 and the band-width of the filter.

An example of a band-pass filter is shown in Fig. 30.

Taking a band width of 120 c/s, a mid-band frequency of 420 c/s, and an impedance of 600 Ω we have—

$$L = \frac{600}{6\cdot28\ \sqrt{(360 \times 480)}}$$
$$= 0\cdot23 \text{ H.}$$

$$C = \frac{1}{6\cdot28 \times 600\ \sqrt{(360 \times 480)}}$$
$$= 0\cdot639\ \mu\text{F.}$$

$$n = \frac{120}{\sqrt{(360 \times 480)}} = 0\cdot289.$$

$$L/n = 0\cdot796 \text{ H}; \quad Cn = 0\cdot184\ \mu\text{F.}$$
$$Ln/2 = 0\cdot033 \text{ H}; \quad 2C/n = 4\cdot42\ \mu\text{F.}$$

The characteristic impedance of basic-type band-pass filters is shown in Fig. 31.

Band-pass filters having somewhat different attenuation characteristics can be obtained by omitting any one of the four elements composing the impedances Z_1 and Z_2 of the basic type filter.

Derived Filter Sections. Variations from the basic filter sections can be used to obtain special attenuation characteristics or to obtain an input impedance which is practically
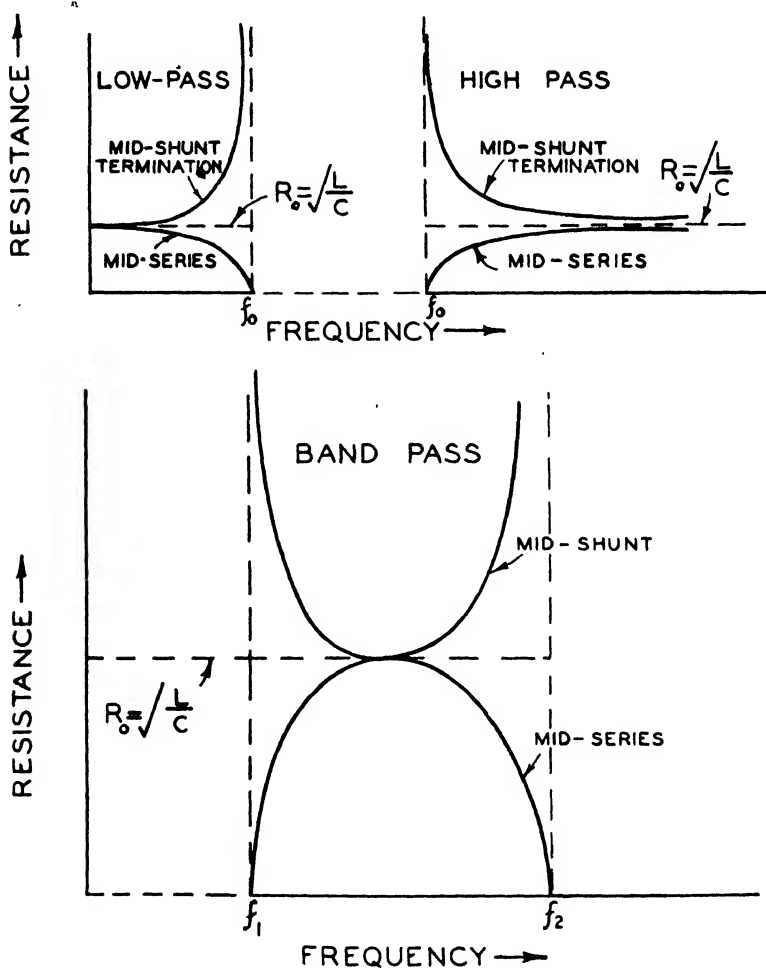

FIG. 31. CHARACTERISTIC IMPEDANCE IN PASS-RANGE OF BASIC-TYPE FILTERS

constant throughout the pass band of the filter. The derivation of such sections is however beyond the scope of the present treatment.

Design and Construction of Filters. It has already been

shown how the value of inductance and capacitance required for any given case can be calculated.

The lower the losses in the components, the lower will be the attenuation in the pass range and the higher the attenuation in the attenuating range. The inductances should therefore have as low an effective resistance as possible consistent with cost and size, and mica di-electric condensers are generally used to obtain low losses and constancy.

The attenuation of a filter is proportional to the number of sections, and two or three sections may be used if a high attenuation is required in the attenuating range.

Inductances. The usual type of inductance coil used for high quality audio-frequency filters are dust-cored toroidal coils similar to loading coils. By this means the variation of inductance with current and frequency is kept to a low value, while the losses also are relatively small.

For low frequency filters such as the low-pass filters used in sub-audio telegraphy where large inductances are required, and which have to carry the fairly large telegraph currents, cores consisting of high permeability stampings (mumetal, permalloy, radiometal, or rhometal) give the best results.

The inductances in narrow-band voice-frequency filters must be constructed to a high degree of accuracy, the accuracy specified usually being $\pm$ 1 per cent of the correct value.

The inductances used in the filters on the multi-channel voice-frequency telegraph system used in the British Post Office are wound on permalloy dust cores.

Condensers. For stability and low losses, mica dielectric condensers meet all the requirements for audio- and carrier-frequency filters. They are usually constructed to an accuracy of $\pm$ ½ per cent of the correct value.

For low frequency low-pass filters used for sub-audio telegraphy, accuracy is not so important, and as large value condensers may be required (over 10 μF), paper dielectric condensers are used since the space and cost per microfarad is much less than that of mica condensers.

BAND-PASS FILTERS USED IN THE MULTI-CHANNEL VOICE-FREQUENCY TELEGRAPH SYSTEM

The nominal band-width of the filters used in the multi-channel voice-frequency system used in the British Post Office is 120 c/s. The receiving filters are designed so that the attenuation $\pm$ 35 c/s from the mid-band frequency is not

more than 2·5 db above that at the mid-band frequency. The attenuation $\pm$ 120 c/s from the mid-band frequency (i.e. at the carrier frequency of the adjacent channels) is at least 30 db greater than that at the mid-band frequency.

The filters are directly made up from the basic filter section. However, to obtain more convenient values for the shunt inductance and condensers, an internal impedance transformation is effected by the use of an additional shunting condenser connected between the series inductance and condenser.

Balanced and Unbalanced Filters. Filters are usually made up as unbalanced networks to economize components. In such cases it is desirable that the short-circuit side of the filter be at earth potential. If balanced conditions are required, they are frequently obtained by interposing a transformer between the unbalanced filter and the balanced circuit.

Uses of Filters. The following are amongst the many uses of filters in telecommunications—

Low-pass filters having a cut-off frequency of about 140 c/s are inserted in the sending end of direct current telegraph circuits which work in the same cables as telephone circuits so as to eliminate the audio-frequency components of the telegraph signals to prevent noise in the telephone circuits. The use of such a filter enables single-wire telegraph circuits to be worked in telephone cables without any appreciable noise occurring in adjacent telephone circuits. Normally a single-section filter is used, suitable values for the inductances being 1·35 H and for the condenser 2 μF.

Low-pass filters are also used in a similar manner on sub-audio telegraph circuits.

In supra-acoustic telegraphy a low-pass filter is used to prevent the telegraph signals reaching the telephone terminals on the common transmission circuit.

High-pass filters are used to segregate the telephone and telegraph channels on a sub-audio telegraph system. They are also used to prevent the telephone speech currents reaching the telegraph receiver on a supra-acoustic telegraph system.

Band-pass filters are mainly used in multi-channel systems both telegraph and telephone.

SECTION 6

TRANSMISSION OF A.C. TELEGRAPH SIGNALS

Effect of Build-up Time. It has already been shown in connexion with d.c. telegraph transmission how the quality of received telegraph signals is dependent upon the transient time, that is, the time required for the arrival curve to build up

(a) D C. SIGNALS .   (b) MODULATED CARRIER FREQUENCY WITH SQUARE ENVELOPE
(c) SQUARE ENVELOPE SIGNALS RECTIFIED
(d) MODULATED CARRIER SIGNALS AFTER TRANSIENT EFFECTS HAVE BEEN INTRODUCED
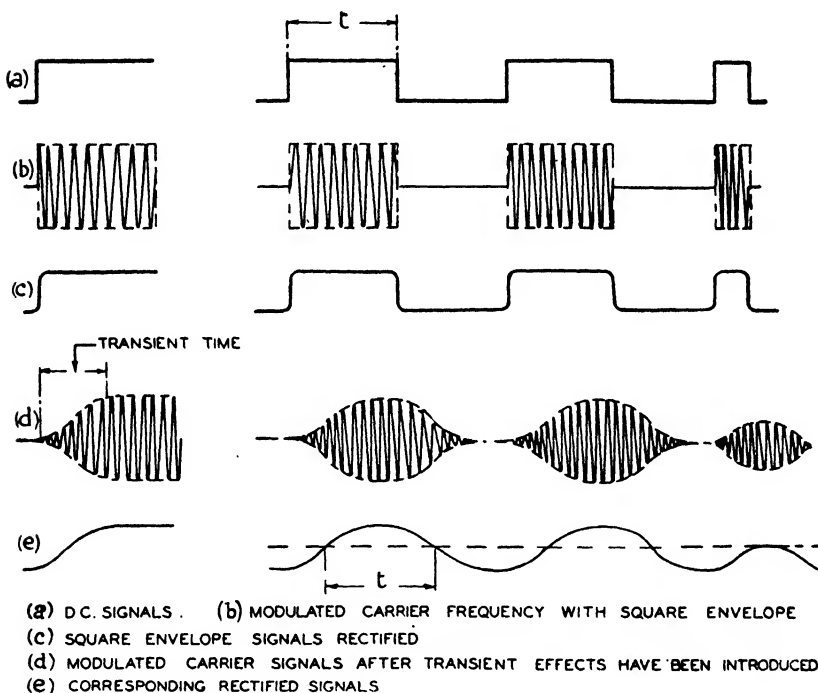(e) CORRESPONDING RECTIFIED SIGNALS

FIG. 32. D.C. TELEGRAPH SIGNALS AND CORRESPONDING A.C.
AND RECTIFIED A.C. SIGNALS

to the steady current value. Exactly the same conditions apply in a.c. telegraphy, when an alternating carrier current is modulated, or interrupted, in accordance with the telegraph code, and subsequently rectified to produce the direct current signals necessary to operate the receiving relay or telegraph instrument. This is illustrated in Fig. 32, where d.c. signals and the corresponding a.c. and rectified a.c. signals are shown.

Any ripple in the rectified signal due to the carrier frequency is assumed to have been smoothed out.

It will be seen that the shape of the rectified signal conforms to that of the envelope of the carrier signal. If the carrier signals are transmitted through a line or network which introduces transient effects the build-up of the signal will be prolonged and the rectified signal will be delayed in building up to the steady value. So long as the build-up time is not greater than the duration of the shortest signal element, a properly biased receiving relay will give out signals of the same duration as the sent signals. It will be appreciated that to respond to the rectified signals the receiving relay must be biased either mechanically or electrically so that the bias has exactly the same effect as a current half the magnitude of the operating current. The biasing point is shown dotted in Fig. 32, and it is obvious that if the relay were biased above or below this line the output signals at the relay contacts would be too short or too long as the case may be.

It is apparent that the relation between build-up time and telegraph distortion in a.c. transmission is exactly the same as that in d.c. transmission, as already explained and illustrated in Fig. 2. The effect is further illustrated in Fig. 32, in which it is shown that if the duration of the sent signal is appreciably less than the build-up time the signal will have no effect on the receiving relay. If a train of such short signals (spacing and marking elements being equal) were sent, the received carrier frequency and rectified current would build up to half the normal value and remain close to this value until the signals ceased.

Frequencies Present in Amplitude Modulated Carrier. It is apparent that the rectified signals shown at (c) in Fig. 32 must have a similar frequency spectrum to the original d.c. signals, and that frequencies corresponding to these must have been transmitted with the carrier.

The frequencies present in square-topped reversals were illustrated in Fig. 16 and given mathematically in equation (2). The case for similar single current signals is illustrated in Fig. 33, and the equation is obtained from equation (2) by adding a constant term to displace the negative side of the signals to the zero line as follows—

$$y_1 = (a/2) + (2a/\pi) [\sin \omega_s t + \tfrac{1}{3} \sin 3 \omega_s t \ . \ . \ .]$$

where $a =$ amplitude of single current signals;

$$\omega_s = 2\pi/T = 2\pi f_s;$$

f_s = number of signal pulses per sec.

The expression for the modulated carrier frequency is
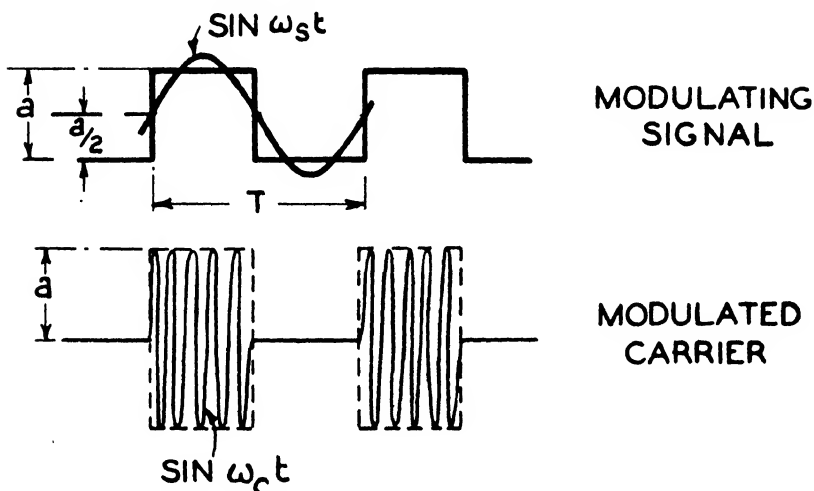
$$y_2 = y_1 \sin \omega_c t$$

SIN ω_st

a

a/2

T

MODULATING SIGNAL

a

SIN ω_ct

MODULATED CARRIER

FIG. 33. CARRIER FREQUENCY MODULATED BY PERIODIC SQUARE-TOPPED SIGNALS

Where y_1 represents the amplitude of the carrier; and

$$\omega_c/2\pi = f_c = \text{frequency of carrier.}$$

We have therefore—

$$y_2 = (a/2) \sin \omega_c t + (2a/\pi) \sin \omega_c t \, [\sin \omega_s t + \tfrac{1}{3} \sin 3\omega_s t \ldots]$$

It can easily be shown that this expression is identical with the following—

$$y_2 = (a/2) \sin \omega_c t + (2a/\pi) \, [\sin (\omega_c + \omega_s)t + \sin (\omega_c - \omega_s)t$$
$$+ \tfrac{1}{3} \sin (\omega_c + 3 \omega_s)t + \tfrac{1}{3} \sin (\omega_c - 3\omega_s)t \ldots].$$

Therefore the modulated wave consists of a series of sine wave components with the following frequencies—

Lower Side Band	Carrier	Upper Side Band
$f_c - f_s$		$f_c + f_s$
$f_c - 3f_s$	f_c	$f_c + 3f_s$
$f_c - 5f_s$		$f_c + 5f_s$
etc.		etc.

It will be seen that any frequency present in the modulating signals is reproduced in the modulated signal as two frequencies represented by the sum and difference of the carrier and signal frequency. These frequencies make up what are termed the *upper* and *lower* side-bands. It follows therefore that signals containing a band of frequencies from zero up to any frequency f_0 will produce frequencies ranging from $f_c - f_0$ to $f_c + f_0$ when used to modulate a carrier frequency f_c, and the frequency band will now be $2f_0$. As in the case of a.c. telegraph transmission, it is necessary to transmit both side-bands: it will be seen that the frequency band required is twice that for d.c. signals.

Frequency Band Required. In multi-channel a.c. telegraphy, carrier waves of different frequencies are used for each channel, and the signals segregated by means of band-pass filters at the terminals of the common transmission line. It is obvious that the narrower the band of frequencies occupied by each channel, the more are the channels which can be operated on a transmission line capable of transmitting a given range of frequencies. In a properly designed system therefore the band-width of the channel filters will be no wider than is necessary for the speed of signalling required.

When square-topped modulated signals such as those shown in Fig. 33 are passed through a band-pass filter, the effect of the consequent curtailment of the side-bands is to delay the building up of the signal envelope in a manner somewhat like that shown at (*d*) in Fig. 32.

It can be shown that for a band-pass filter, the time required for a carrier at the mid-band frequency to build-up to the steady value is approximately equal to the inverse of the effective band-width.

$$\text{Thus } t \doteqdot 1/(f_2 - f_1),$$

where $t =$ build-up time in sec.; and

$$f_2, f_1 = \textit{effective} \text{ cut-off frequencies.}$$

It follows that signals having a duration of less than t sec. will not be transmitted properly, and therefore the limiting telegraph speed in bauds is given by—

$$N = 1/t = f_2 - f_1,$$

where N, the speed in bauds, is given by the inverse of the shortest signal element in seconds.

There is therefore a very simple relationship between frequency band required and the speed of signalling. It has been shown that the frequency band required for a modulated carrier frequency is twice that of the modulating or d.c. signals, and therefore the relationship between signalling speed and frequency band can be stated as follows—

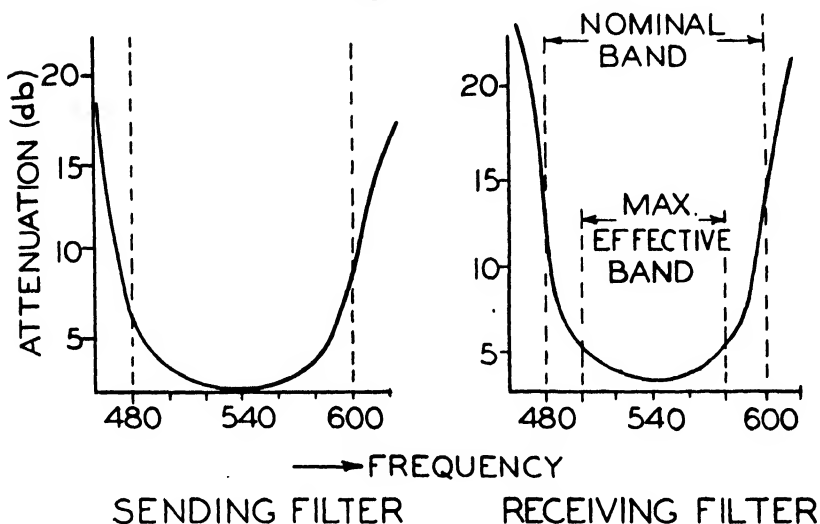The minimum band of frequencies required for d.c. telegraph

FIG. 34. TYPICAL ATTENUATION CHARACTERISTICS OF FILTERS OF MULTI-CHANNEL V.F. SYSTEM

signals is equal to half the telegraph speed in bauds, and for modulated a.c. signals it is equal to the speed in bauds.

It follows from this that, other things being fixed, the effective band-width required in any given case is directly proportional to the speed of working required. It does not follow however that the number of channels obtainable on a given transmission line is inversely proportional to the speed at which the channels are to be worked.

Referring to Fig. 34, it will be seen that the maximum effective band-width of the receiving filter is approximately 80 c/s, giving a maximum speed of signalling of approximately 80 bauds. The fundamental frequency of reversals at a speed of 80 bauds is 40 c/s, and therefore if the carrier frequency of 540 c/s is modulated by this frequency, the side-band frequencies will be 580 and 500 c/s.

At higher speeds the amplitude of the side-band frequencies

would be attenuated and eventually the signals would be imperceptible at the receiving end.

In practice, distortion due to other effects, such as external interference, bias, variation of transmission level, are present, and it is not satisfactory to attempt to work channels, either d.c. or a.c., too near the limiting speed. Also in the case of a.c. transmission, any variation of the carrier-frequency from the mid-band frequency is more serious when working at high speeds.

It will be appreciated therefore that the speed of working is not the only factor which determines the frequency band per channel, and therefore the number of channels which can be obtained in any given case.

Factors Affecting the Frequency Spacing for Multi-Channel A.C. Telegraphy. The various factors which determine the number of channels obtainable on a multi-channel system are as follows—

(*a*) The speed of working required.

(*b*) The ratio of effective to nominal band-width which can be realized in the practical filter.

The smaller the band-width with respect to the mid-band frequency, the more difficult it is to secure a good ratio.

(*c*) The variation in transmission level required to be met without re-adjustment.

The narrower the band-width the smaller the variation in level that can be satisfactorily tolerated.

(*d*) The variation in carrier frequency to be tolerated.

(*e*) The level of the interference currents reaching a detector with respect to the normal signal currents.

Such interference currents may be due to inter-modulation products (harmonics) caused by loading coils and repeaters in the transmission line. They may also be due to external interference or cross-talk.

(*f*) The signal distortion permissible in practice.

The standard frequency spacing adopted for multi-channel voice-frequency telegraph systems is 120 c/s. This enables 18 channels to be worked on a telephone trunk circuit which has a frequency range of 300 to 2 600 c/s.

This standard is based on the use of high-grade trunk circuits on which the cross-talk level is low, and the repeaters are capable of giving an output of 50 mW with very little distortion.

The highest quality magnetic material is also used for the

cores of the filter inductances, an effective band-width of 80 c/s being secured.

This choice of frequency spacing constitutes a compromise amongst all the relevant factors, and enables a high quality telegraph service to be provided on existing trunk circuits without any waste of the available frequency spectrum.

The majority of the channels are used for teleprinter working, that is at a telegraph speed of 50 bauds. At this speed the distortion of the channels under normal conditions is usually about 6 per cent and rarely greater than 10 per cent. This allows an ample margin in service to cover various contingencies.

As already stated, the band-width of 80 c/s permits the channels to be worked up to 80 bauds. At this speed of working, however, the margin of safety is very much reduced and it would be uneconomic to attempt to work other than a few special channels at this limiting speed.

A further factor affecting the minimum permissible band-width for the channels is the need for setting up through connexions by means of voice-frequency channels connected in tandem. Teleprinters may in such cases be required to operate over three or four channels connected in series. The cumulative effect of the distortion contributed by each channel would render this impossible at speeds approaching 80 bauds, but it is quite permissible at the normal teleprinter speed of 50 bauds. The question of tandem working is dealt with in more detail in the section entitled "Transmission Requirements for Telegraph Switching Systems," page 96.

Line Requirements for Multi-channel Systems: Inter-channel Interference and Cross Modulation. The transmission qualities necessary for the main line used for a multi-channel voice-frequency telegraph system are essentially the same as those of a high quality trunk-telephone circuit. The attenuation and phase characteristics of a line of the type used are shown in Fig. 19. On some lines it may be necessary to use an attenuation equalizer to secure a uniform attenuation over the band of frequencies required, that is from 60 c/s below the bottom channel frequency to 60 c/s above the top channel frequency. It is important that the received level for any channel should not be appreciably lower than that for other channels, particularly the adjacent channels, otherwise the channel is liable to be affected by interference currents from other channels.

Non-linearity of the phase characteristic is of secondary importance, as a difference of phase between channels is of no

importance and a channel band itself is so narrow that no appreciable relative phase displacement occurs over this small range. This is fortunate since equalization of phase is more difficult on long loaded lines than equalization of attenuation.

Noise on the main line would need to be rather worse than that normally expected on telephone lines before interference would be caused to the telegraph signals.

The remaining factor effecting M.C.V.F. telegraph operation is cross modulation. This arises from amplitude distortion of the wave form due to the presence of non-linear components in the line, that is, components in which the ratio of voltage to current is not constant, but dependent on the value of the voltage and/or its direction. Iron-cored inductors and transformers are non-linear due to curvature of the B/H curve, the effect being that the magnetic flux is not absolutely directly proportional to the current and the inductance varies as the current rises and falls. The effect is independent of the direction of the current and is, therefore, the same for positive and negative half-cycles of the alternating current.

A similar effect is produced by curvature in valve characteristics, but in this case a rectifying action can also be present due to the positive half-cycle being amplified to a greater extent than the negative half-cycle, or vice versa.

This non-linearity is productive of harmonics, that is, spurious frequencies, which are multiples of the applied frequencies. (Symmetrical distortion of the positive and negative half-cycles produces odd harmonics while asymmetrical distortion (rectifying effect) produces even harmonics.) This is illustrated in Fig. 35 in which is shown a peaky, but symmetrical wave which is equivalent to a sine wave with a third harmonic, and an asymmetrical wave which is equivalent to a sine wave with a second harmonic.

In practice the use of very high quality iron-dust cores reduces non-linearity effects in transformers and loading coils to a very low order, and the valve characteristics are such that even harmonics are the most important result of non-linearity.

When more than one frequency is transmitted over the line the effect is somewhat more complicated. If there are two frequencies present these will beat together and the combined effect is to produce a wave whose amplitude varies at a frequency equal to the difference between the two orginal frequencies. This is illustrated in Fig. 36. The resultant is somewhat similar to an amplitude modulated wave, and any rectifying
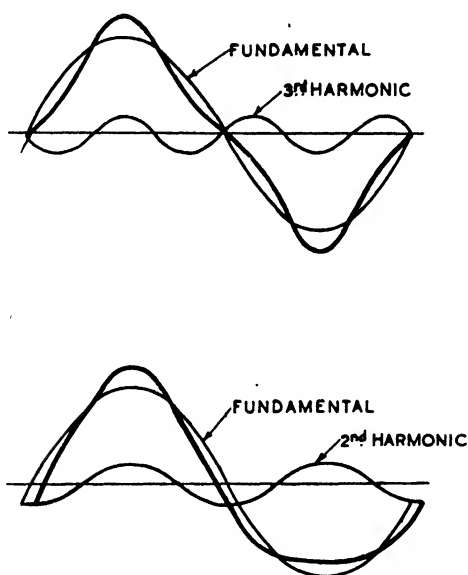
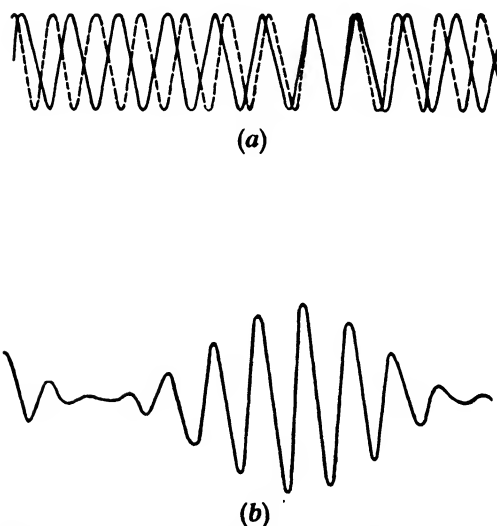FIG. 35. PRODUCTION OF HARMONICS BY DISTORTION OF WAVE-FORM

(a)

(b)

FIG. 36. EFFECT OF ADDING TWO WAVES OF DIFFERENT FREQUENCIES

(a) Applied E.M.F.'s   (b) Resultant wave-form.

action in the valves will cause frequencies equal to the difference and sum of the two original frequencies to appear at the output. The fact that both sum and difference frequencies are produced will be clear from analogy with the normal case of amplitude modulation, the modulation of a frequency f_2 by a frequency f_1 producing the side-band frequencies $f_2 - f_1$ and $f_2 + f_1$. Then if both odd and even harmonics are produced, when two frequencies f_1 and f_2 are transmitted together the harmonics $2f_1$, $3f_1$, $4f_1$, etc., and $2f_2$, $3f_2$, $4f_2$, etc., will be produced together with all the sum and difference frequencies resulting from the beating together of these frequencies; which gives $f_1 - f_2$, $2f_1 - f_2$, $2(f_1 - f_2)$, etc., also $f_1 + f_2$, $2f_1 + f_2$, $2(f_1 + f_2)$, etc.

Thus multiples of the original frequencies are produced together with the sum and difference of the frequencies and their multiples. In practice asymmetrical distortion is usually greater than symmetrical distortion; the distortion products are therefore mainly the even harmonics and the resulting sum and difference frequencies. The first order products $2f_1$, $2f_2$, $f_1 - f_2$ and $f_1 + f_2$ are those having the greatest amplitude, while harmonics having frequencies outside the range of the V.F. system are of course of no account.

The choice of odd multiples of a common frequency for the channel frequencies in M.C.V.F. telegraphy means that even harmonics and sum and difference frequencies tend to fall between the channels where they cause less disturbance. If too high a level of transmitted power is used for the channel frequencies, overloading of the valves will occur and serious distortion of the wave-form will then result; the level is therefore maintained at a value well within the output capacity of the repeater valves.

Cross modulation effects are reduced when feed-back type amplifiers are used. Since the effect of the feed back is to maintain the gain constant for all values of signal amplitude, amplitude distortion is reduced to very small proportions.

Cross modulation products arising at various points in the line are not necessarily additive. Owing to the relative changes in phase of the various frequencies due to phase distortion, spurious frequencies produced by one repeater are unlikely to be of the same phase as corresponding frequencies produced by some earlier repeater. The combined effects for any particular spurious frequency are therefore not likely to be directly additive and may even result in a decrease.

In practice so long as attenuation equalization is adequate, the interference products are at such a low level that the performance of V.F. channels is not seriously affected by the length of the line.

Effect of Variation in the Level of the Received Signals. The ordinary method of voice-frequency telegraph working

A.C. SIGNALS   RECTIFIED SIGNALS
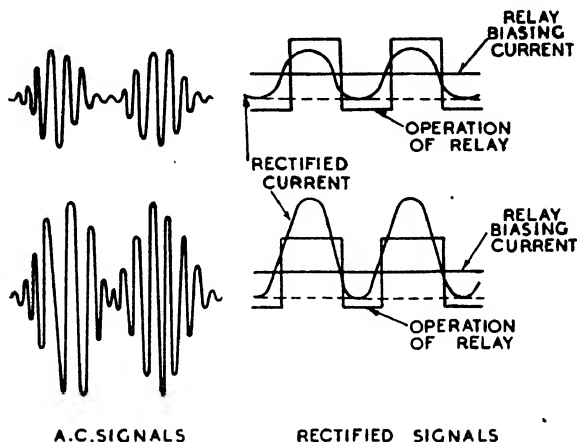
FIG. 37. PRODUCTION OF BIAS BY CHANGE IN AMPLITUDE

by keying a tone is analogous to single current working in that the receiving relay has to be biased to an extent equivalent to half the amplitude of the received signal. If the amplitude of the signals operating the relay varies, then signals given out by the relay will show a bias. This is illustrated in Fig. 37, which shows the bias produced in the signals if the amplitude of the rectified signals increases but the relay biasing current remains unchanged. In practice the effect is reduced by means of automatic gain control in the amplifier-detector, and with "tone-on" working a range of level of the received signal of up to 15 db can readily be accommodated, and with special arrangements up to 40 db can be catered for. With "tone-off" working the range is reduced to 6 db. With tone-on working, where signalling is by interruption of an otherwise steadily transmitted tone, the gain control must remain relatively steady during the longest break in tone which occurs during signalling. Even with the largest practicable time-constant for the gain control circuit some recovery is bound to occur when no tone is present to maintain the biasing voltage, and

a limit is, therefore, set for the range of permissible level variation. With tone-off working, the tone is normally cut off and only transmitted during signalling. During idle periods the amplifier returns to maximum sensitivity and thus, during the commencement of the signal, the gain control does not come into operation.

Fig. 38 illustrates the case where the rectified signal has
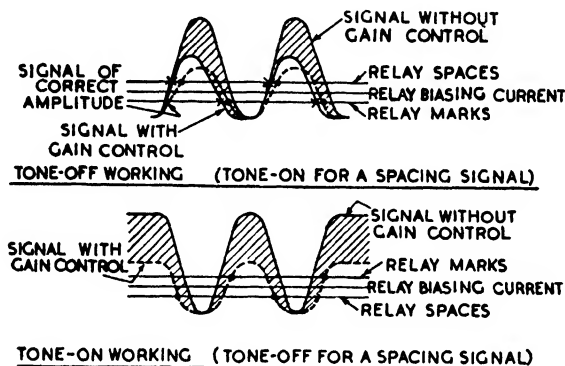
FIG. 38. GAIN CONTROL ACTION IN VOICE-FREQUENCY TELEGRAPHY

twice the amplitude of that required to give undistorted signals, therefore corresponding to a level change of 6 db. In the tone-on case the effect of the gain control is to bring the rectified signal to the correct amplitude before signalling commences, and little change results in the signal shape, which remains reasonably symmetrical. In the tone-off case, no effect is produced until the signal rises above the value at which the gain control comes into operation; thus the relay operates to spacing sooner than it would on a signal of smaller amplitude. Unless the time at which the relay operates to marking is also advanced to correspond, spacing bias will result.

The reduction in amplitude necessary to achieve this effect is indicated by the dotted line in the figure and corresponds to a reduction in gain at this instant of something more than twice the actual change in level of the signal.

As soon as the signal ceases the gain must rapidly return to normal so that when the next signal arrives the amplifier is in the same condition as it was when the first signal arrived. It is clear that this calls for a much more critical performance of the gain control circuit than with tone-on working, and a smaller range of permissible level variation naturally results.

The effects have been illustrated with respect to the signal shapes when signalling at normal speeds on voice-frequency channels. It is clear that for squarer signals, that is, signals with a shorter build-up time, such as would be obtained by widening the band-width, the effect of level variation would be less serious. This explains why, in the system of teleprinting over telephone exchange connexions (usually referred to as "Telex") where the whole telephone-frequency band is available for one channel, a wide range of level variation is possible. Although it is necessary to have tone-off in the idle condition,
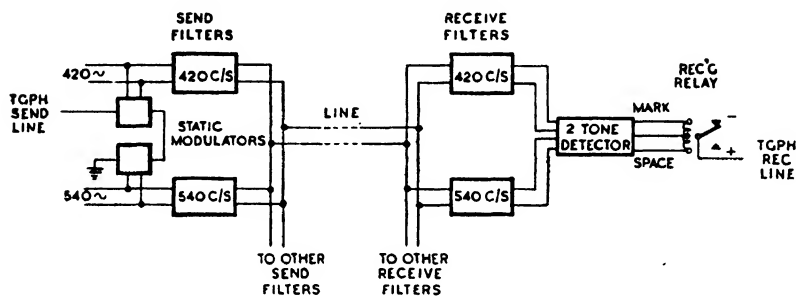
FIG. 39. BLOCK SCHEMATIC OF TWO-TONE TELEGRAPHY

because each way working on a single pair of wires is required it is possible to correct for level variations of up to as much as 35 db.

Tone-on working is used on all multi-channel voice-frequency telegraph systems in the British Post Office, and has also been standardized for international services.

Multi-tone Working. When multi-channel voice-frequency telegraph systems are operated over radio links or very long overhead lines, large changes in level may be experienced. On a short-wave radio link in particular, the effect of fading is such that the level of the received signal is continuously changing. To meet these conditions two-tone working can be used in which two adjacent channels are combined to form one circuit, and arranged so that one channel transmits tone for a mark and the second channel tone for a space. The receiving relay is arranged to be under the control of both the mark channel and the space channel, giving the effect of double current working. A diagram of one arrangement is given in Figs. 39 and 40.

When marking tone is received it is amplified by V_1 and rectified by the full-wave rectifier V_3. The rectified signal

appears as a positive bias on the control grid of the output pentode V_5, causing anode current to flow. This current passes through the marking coil of the receiving relay, the armature of which is operated to the marking contact. R_L and C_L act as a grid limiter to avoid undue increase of anode current when the level of the received signal rises; they also ensure full suppression of the spacing output valve V_6 when a marking signal is being received. When the signal voltage applied to V_5 exceeds the fixed negative bias of 3 volts, grid current will flow and develop a voltage across R_L and reservoir condenser C_L,
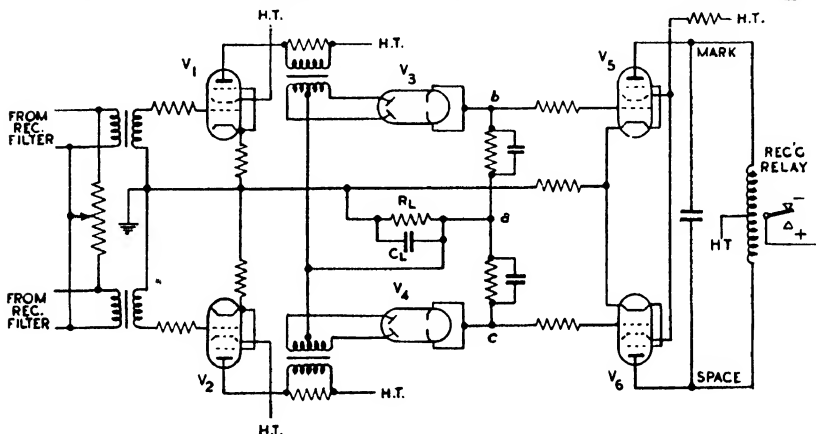
FIG. 40. TWO-TONE DETECTOR

this voltage being such as to prevent the grid voltage rising above the point where grid current just commences to flow. Since this bias voltage is also applied to the grid of V_6, which is receiving no signal at this stage, any stray noise being received through the "space" filter is prevented from affecting the marking signal.

The time constant of C_L, R_L is nominally 10 msec and is sufficiently small to allow the detector to readjust itself quickly when rapid variations in input level occur. When the signal changes to spacing, control of the limiter is taken over by the spacing signal, the anode current in V_5 ceases, and anode current in V_6 operates the relay to spacing.

With this detector, a range of input level approaching 50 db is possible without readjustment.

The potentiometer connected across the mark and space inputs is provided to adjust for slight inequalities between the valves and components on the two sides of the detector.

When fading conditions are severe, four-tone working can be used. Two "two-tone" channels are used in parallel giving two mark frequencies and two space frequencies. During marking periods the mark frequency which is being received at the higher level will take charge of the output valve, and similarly the stronger signal takes charge during spacing periods. Arranging, in this way, for the stronger signal to take charge, prevents noise associated with the weak signal from affecting the output. The type of fading experienced on short-wave radio circuits is such that usually both frequencies will not experience severe fading at the same time and thus the effect of fading is much reduced.

The method of coupling the detector panels of the two two-tone circuits is as follows. The output pentodes and receiving relay of the second channel are not used, and the output of the mark and space diodes is connected across points a and b (mark diode) and a and c (space diode) of the first detector. Both mark signals, are therefore applied to V_5 and both space signals to V_6.

The type of fading whereby one part of the frequency band fades while the remainder of the band is received at a satisfactory level is termed *selective* fading, and the method using two or more tones together by paralleling channels is called *frequency diversity*. Another method of overcoming fading which can also be used is *space diversity*. In this case two or three radio receivers are used each having a separate antenna system, the antennae being so spaced that a given frequency is not likely to fade on all receivers simultaneously. A complete multi-channel voice-frequency telegraph receiving equipment is connected to each receiver, the detectors of each channel being paralleled in the same way as for frequency diversity. If three receivers are used the outputs of the diodes of the second and third detectors are both paralleled across the points a and b and a and c of the first detector.

FREQUENCY MODULATION

The foregoing paragraphs have dealt with amplitude modulation of an a.c. carrier wave, and this is the method almost exclusively used for a.c. telegraphy on wire lines. Attention has in recent years been given to an alternative method of modulation, namely frequency modulation (F.M.). In frequency modulation, instead of the modulating signal controlling the amplitude of the carrier it controls its frequency. The effect

of the modulating signal is to vary the frequency of the carrier above and below the mean value by an amount depending upon the amplitude of the modulating signal, as illustrated in Fig. 41. An important advantage of frequency modulation as compared with amplitude modulation is that a reduction of the effect of noise or interfering currents in the transmission link can be secured. In amplitude modulation such currents have the effect of varying the amplitude of the transmitted wave, and are thus reproduced together with the signal in the rectified output of the receiver. With frequency modulation, although the interfering currents still vary the amplitude of the carrier, these variations in amplitude can be eliminated by means of a limiting stage in the receiver prior to demodulation. Though this does not completely remove the effect of noise, a marked improvement in signal/noise ratio can frequently be obtained.

MODULATING SIGNAL

AMPLITUDE MODULATED CARRIER

FREQUENCY MODULATED CARRIER

FIG. 41. COMPARISON OF AMPLITUDE AND FREQUENCY MODULATION

The foregoing characteristic of frequency modulation is primarily of importance in connexion with the transmission of speech and music by radio on wideband channels. Conditions are rather different when the problem is that of transmitting telegraph signals on relatively narrowband channels. In this case, noise is of less consequence, particularly on land lines; on the other hand, with frequency modulation the effect of level variations can be more accurately compensated for than with amplitude modulation.

Since the modulated signal wave sent out by the transmitter has a constant amplitude, it can be passed through a limiter at the receiver which removes all changes in amplitude produced in the transmission link, so long as the level does not fall below that at which limiting takes place.

Very frequently the lines used for multi-channel voice-frequency are contained in underground cables and are thus relatively stable and little subject to noise. However, where systems are operated over long overhead lines or radio links, greater variations in signal strength and higher noise levels are encountered, and frequency modulation of the channel

frequencies by the telegraph signals has some advantage over amplitude modulation. In telegraphy frequency modulation has, however, the disadvantage as compared with amplitude modulation that the modulating, demodulating and carrier-frequency generating circuits are more complex, and accuracy of the carrier frequency is of greater importance.
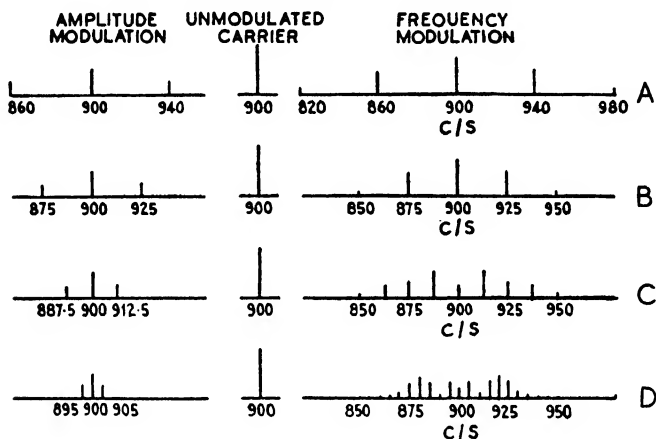
FIG. 42. AMPLITUDE OF CARRIER AND SIDE FREQUENCIES IN MODULATED WAVES

A. $f_m = 40$ c/s (80 bauds), $f_c = 900$ c/s, $\triangle f = 40$ c/s, $\triangle f/f_m = 1$.
B. $f_m = 25$ c/s (50 bauds), $\triangle f = 25$ c/s, $\triangle f/f_m = 1$.
C. $f_m = 12\cdot5$ c/s, $\triangle f = 25$ c/s, $\triangle f/f_m = 2$.
D. $f_m = 5$ c/s, $\triangle f = 25$ c/s, $\triangle f/f_m = 5$.
(*Note.* The amplitudes given for A.M. are for "on-off" keying as used in telegraphy, where the peak amplitudes of the modulated and unmodulated waves are the same.)

Frequency Band Required for F.M. Telegraphy. A frequency modulated carrier can be resolved mathematically into carrier and side-band components, but the result is considerably more complicated than that for amplitude modulation.

For the simple case of modulation by a single sine-wave if

f_m = modulating frequency;

f_c = carrier frequency;

$\triangle f$ = frequency deviation or swing either side of the mean carrier frequency,

the modulated carrier will consist of a frequency f_c with side frequencies of

$$f_c + f_m, f_c - f_m, f_c + 2f_m, f_c - 2f_m, f_c + 3f_m, f_c - 3f_m. \cdots$$

and so on to infinity. In theory, therefore, there is an infinite number of side frequencies spaced at intervals equal to the

modulating frequency. However, the energy content of side frequencies lying outside the frequencies

$$f_c + f_m + \triangle f \text{ and } f_c - f_m - \triangle f$$

is very small, and the effective band-width of the modulated signal is therefore equal to 2 $(f_m + \triangle f)$ where f_m is the modulating frequency and $\triangle f$ the frequency displacement obtained when the modulating signal is at maximum amplitude.

This is illustrated in Fig. 42, where the relative amplitude of the carrier and side frequencies for typical sine-wave modulations are shown. The corresponding conditions for amplitude modulation are also given for comparison.

The small amplitude of side frequencies more than $(f_m + \triangle f)$ cycles per second from the mean carrier frequency will be observed from the figure, and the fact that the greater part of the energy in the modulated wave is within the band 2 $(f_m + \triangle f)$ will be clear from the following.

The energy content of any frequency being proportional to the square of the amplitude, we have:

Energy distribution in frequency modulated carrier ($\triangle f/f_m = 1$)	Relative Amplitude	Relative Energy
Unmodulated Carrier (f_c)	1	1
f_c	0·76	0·5776
$f_c + f_m$	0·44	0·1936
$f_c - f_m$	0·44	0·1936
$f_c + 2f_m$	0·11	0·0121
$f_c - 2f_m$	0·11	0·0121

Sum of carrier and side-frequencies = 0·9890

When a carrier frequency is modulated by a single sinusoidal frequency the ratio of the frequency deviation ($\triangle f$) to the modulating frequency (f_m) is called the *modulation index.*

When a carrier frequency is modulated by mixed signals the conditions are rather more complicated, since each frequency present in the modulating signal contributes something to the total frequency deviation. The term *deviation ratio* is used in this case and denotes the ratio of the maximum carrier-frequency deviation to the maximum frequency present in the modulating wave. The maximum deviation occurs, of course, when the modulating wave is at maximum amplitude.

The relative amplitudes of the carrier- and side-frequency components for single-frequency modulation are dependent on

the modulation index. In the first two cases illustrated in Fig. 42, the modulation index is unity and the relative amplitudes of the carrier and side frequencies are the same in each case, although, of course, the actual frequencies are different. With increase of modulation index the number of side frequencies of significant magnitude is greater, although if the frequency deviation remains unchanged the effective band-width is reduced, being equal, as already stated, to twice the sum of the frequency deviation and the modulating frequency.

In telegraphy, when it is desired to restrict the band-width of the channels so as to obtain the maximum number of channels on one line, it is possible without undue increase in distortion to use a band not much greater than that necessary to cover the actual frequency deviation of the carrier frequency, provided the maximum keying frequency does not exceed the frequency deviation or, in other words provided the *deviation ratio* is not less than unity. (It will be observed that the term *deviation ratio* is used in this case since we are concerned with the maximum frequency present in mixed signals and not simple sinusoidal modulation.) Taking the first case illustrated in Fig. 42, where the modulating frequency is 40 c/s and therefore equal to a signalling speed of 80 bauds, restriction of the band-width to, say, 90 c/s would mean that only the first pair of side frequencies would be transmitted. These frequencies are, in this instance, the same as those which would be obtained with amplitude modulation, although, of course, their amplitude and phase with respect to the carrier are different. It will thus be seen that in telegraphy it is possible to adopt frequency modulation and at the same time use a band-width no greater than that normally used for amplitude modulation.*

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* It should perhaps be explained that the restriction of band-width cannot be effected without some increase in distortion. Some restriction in band-width could also be permitted in two-tone working, both by narrowing the bands of the filters and also allowing the pass-bands of associated mark and space channels to overlap more than is possible for channels used for separate circuits. The footnote on page 4 explains that a build-up time in excess of the duration of the shortest signal element is theoretically possible without distortion. This fact was ignored in the treatment of the band-width required for ordinary amplitude modulation voice-frequency telegraphy, because if the build-up time exceeded the length of the shortest signal, signals of different length would be of different amplitude, and the automatic gain control would not function properly. Therefore, while in frequency modulation the amplitude changes resulting from restricted band-width can be removed by simple limiting, this is not possible with single-tone amplitude modulation, and restriction of band-width is therefore more objectionable. There are also objections to the use of limiting in two-tone working, and for comparable performances a wider band is required than that for frequency modulation.

The reason for using a deviation ratio of not less than unity is that unless a reasonable amount of frequency swing is employed the signal cannot be satisfactorily demodulated, drift of carrier frequency becomes serious, and also no worth-while improvement over amplitude modulation will be secured.

It will be appreciated that two-tone working and frequency modulation are somewhat similar. Two-tone working has the advantage of using simple modulating and demodulating equipment, but when employing standard channel equipment as used for single-tone amplitude modulation, it uses a band width per channel twice that required for single-tone working. Where neither extreme variations in level nor high noise levels are encountered, frequency modulation can be employed with the band-width restricted to that required for amplitude modulation, and consequently no sacrifice in the number of channels is necessary.

Signalling Conditions. When sending mixed signals the conditions are somewhat different from those for simple sinusoidal modulations, but if the d.c. modulating signals are smoothed by means of a low-pass filter the frequency bandwidth of the F.M. signal is similar to that for sinusoidal modulations. During signalling the frequency will then move up and down smoothly, and, of course, during long signals the frequency will, for the greater part of the signal, remain steady at the mark or space value as the case may be.

The method of producing F.M. signals is to use an oscillator whose tuned circuit contains a component the reactance of which is varied by the applied d.c. modulating current. In this way the resonant frequency of the tuned circuit is varied and therefore that of the oscillator.

A demodulator for F.M. signals is usually called a *discriminator*. The basic circuit of a suitable type is given in Fig. 43. Two circuits, LC_1 and LC_2, tuned to frequencies above and below the mean carrier frequency, are connected in series. When the carrier frequency swings towards the frequency to which LC_1 is tuned, the rectified current in diode V_1 increases and that in V_2 falls, the action being reversed when the carrier frequency swings towards the resonant frequency of LC_2. If the output after amplification is differentially applied to the receiving relay a double current effect is produced. In practice, a limiting stage and a d.c. amplifying stage would be necessary, but for clarity these have not been shown in detail and contain no special features.

It will be clear that any drift of the mean carrier frequency will have the effect of producing a bias. For example, if the frequency drifts towards the spacing side, the output from V_1 will fall and that from V_2 will rise, giving a spacing bias.

In normal V.F. telegraphy, a carrier frequency error of 6 c/s
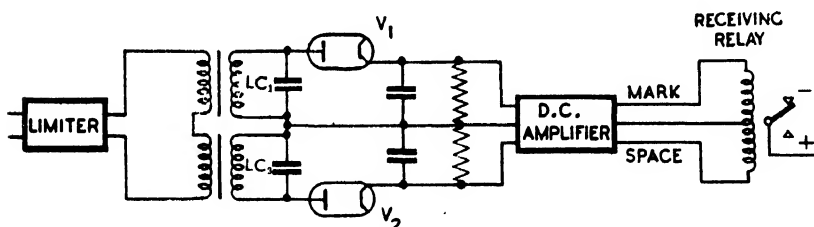
FIG. 43. PRINCIPLE OF DISCRIMINATOR FOR F.M. TELEGRAPHY

is not of serious consequence, but in F.M. telegraphy using a frequency deviation of, say, 30 c/s this would cause appreciable bias. In this respect two-tone working is superior to F.M. This is of particular importance if the multi-channel system is to be operated over a carrier telephone channel, working with suppressed carrier, and in which the re-inserted carrier is not locked to the same frequency as the original carrier. In this case a difference between the two carrier frequencies will cause a similar shift in the telegraph tone frequencies, and if these are frequency modulated the received telegraph signals will be biased.

SECTION 7

TRANSMISSION REQUIREMENTS FOR TELEGRAPH SWITCHING SYSTEMS

Cumulative Effect of Distortion. The standardization of the teleprinter for the majority of telegraph services, and the extensive use of multi-channel voice-frequency telegraphy has led to the widespread introduction of telegraph intercommunication services. In such services, where through connexions may be set up over a number of separate links connected in tandem (i.e. in series) it is necessary that the distortion introduced by each link should be small, or the cumulative effect on the transmitted signals will result in faulty reception. The stability and high quality of service, so easily obtained with voice-frequency channels, meets these requirements and has encouraged development of telegraph switching systems.

The cumulative effect on the distortion of telegraph signals by their transmission over successive links depends on the type of distortion concerned.

Characteristic Distortion. If links each having the same type of characteristic distortion are connected in tandem, then the distortion will, in most cases, be additive. For example, if the distortion is of the type where short mark signals following long space signals are shortened (e.g. as illustrated in Fig. 2) due to the build-up time exceeding the length of the signal, then, when the signal is repeated by the receiving relay into the next link, the short signal having been already reduced in length will suffer even greater distortion; and after several such repetitions may be completely lost. It is clear, therefore, that characteristic distortion must be negligible, or very small, on the individual links forming part of a switching network.

Bias Distortion. If bias distortion is present in tandem-connected links the effect will only be additive if the bias is of the same sign in each link, and there will be a net reduction if the bias in one or more of the links is of opposite sign to the others. For the case of four links in tandem the chance of the bias in all links being the same sign is 1 in 8, and in most cases the resultant total bias distortion will only be about twice that for a single link. While, therefore, the cumulative effect of bias distortion is not so serious as that of characteristic distortion it must, none the less, be kept to a low value because

bias distortion affects every signal element and a high overall value could not be tolerated.

Fortuitous Distortion. This type of distortion in a transmission link arises in the main from interfering currents produced by external disturbing sources; and in multi-channel systems, interference from other channels in the system causes fortuitous distortion.

Since the distortion produced is quite irregular and only a small proportion of the signal elements receive the maximum distortion, and further, a signal changeover may either be advanced or retarded, the chance of any particular signal suffering maximum distortion of the same kind in each link is very small. In the result if, say, four links each giving a maximum of 10 per cent fortuitous distortion are connected in tandem, the resulting total distortion is rarely likely to exceed 20 per cent. The cumulative effect of fortuitous distortion is therefore of less importance in multi-link connexions than that of either characteristic or bias distortion.

It is very important that channels required to be used in multi-link connexions are not worked at speeds approaching the limit imposed by the band-width of the channels. Suppose, for example, that with a signalling speed of 50 bauds, a 20 msec signal has been shortened by 20 per cent when it reaches the third channel in the connexion. It will now be 16 msec in length only, and the effect is as if the signalling speed had been increased to 62·5 bauds. If this higher speed were near the limit of the channel, a big increase in distortion would result.

In the voice-frequency telegraph system used in the British Post Office the limiting speed is well above the signalling speed of 50 bauds which is the teleprinter signalling speed, and an adequate margin is therefore available. For the same reason the characteristic distortion is very small, and when the channels are properly lined up the distortion is almost wholly confined to fortuitous distortion.

Furthermore, provided the overall attenuation frequency characteristic of the line is uniform, the channel distortion is practically independent of the length of line, and for practical purposes all channels on an intercommunication network can be regarded as having identical transmission qualities independent of their length. Voice-frequency channels are therefore admirably suited for use in teleprinter switching networks.

Relation between Teleprinter Margin and Permissible Overall Distortion. The factors determining the maximum distortion

which can be tolerated on a circuit connecting two tele-printers are :

(a) the degree of distortion of the signals given out by the sending teleprinter ;

(b) the margin of the receiving teleprinter ; and

(c) the speed difference between the two teleprinters.

A teleprinter transmitter, if in good order, can be relied on to give signals accurate to within 1 msec—that is, the signal changeovers will occur within 1 msec of the correct time measured with respect to the commencement of the start signal.

The margin of a teleprinter is usually measured by means of artificially distorted signals in which the code elements are displaced with respect to the start signal, the effect being produced by lengthening and shortening the start signal. Some teleprinters are equipped with an orientation device (also known as a *range finder*) whereby the operation of the selecting mechanism can be advanced or retarded with respect to the commencement of the start signal, and which can therefore be used to measure the margin of the machine.

A teleprinter is said to have a margin of 40 per cent when the code elements can be advanced or retarded up to 40 per cent of a unit signal element before errors occur.

Some conditions relating to the margin of a teleprinter and signal distortion are illustrated in Fig. 44, in which time charts for various conditions are given. The shaded strips indicate the periods during the revolution of the receiving cam when the electromagnet must be definitely set to the position required by the signal code if the character is to be correctly received.

The charts relate to the following conditions :

(1) Correctly adjusted teleprinter—equal margins for mark-ing and spacing signals.

(2) Stop position of receiving cam incorrectly set, selecting periods occurring too late.

(3) Receiving teleprinter 6 per cent fast. (Selecting periods become further advanced with each element.)

(4) Correct reception of letter "*R*."

(5) 20 per cent marking bias on received signal. The space-to-mark transitions are advanced 4 msec (20 per cent of 20 msec) with respect to the mark-to-space transitions. Mark-to-space transitions being of the same type as the start signal remain unchanged with respect to the start signal.

(6) Distorted received signals. The second element is not correctly selected and an error results.

The effect of a difference of speed between the sending and receiving teleprinter is illustrated in Fig. 44. If the speed of the teleprinters on a switched network is within 1 per cent of the correct value, the maximum difference which will occur
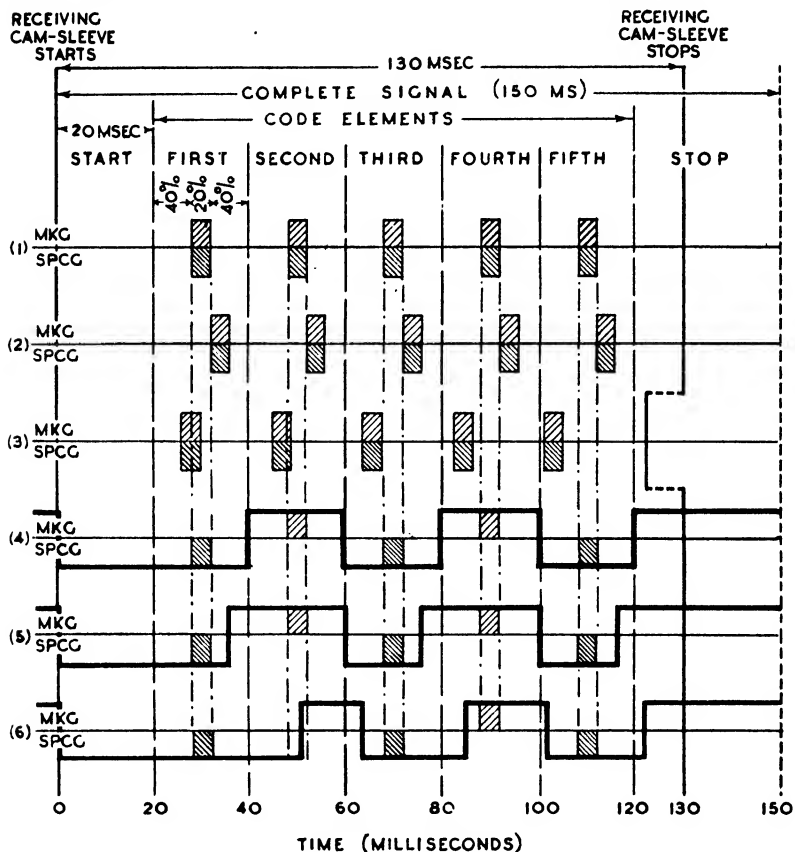
FIG. 44. TELEPRINTER MARGIN AND SIGNAL DISTORTION

when any two teleprinters are connected together will be 2 per cent. Such a speed difference is equivalent to a loss margin of approximately 10 per cent on the fifth code element, and 8 per cent on the fourth.

In considering the maximum distortion permissible on multi-link teleprinter connexions, it must be remembered that the chance of all factors being simultaneously of maximum adverseness is very small. Such a case would occur if, for

example, a sending teleprinter were 1 per cent slow, and at one and the same time:

(a) the transmitter distortion were such that a changeover from space to mark at the commencement of the fifth code element were a full 5 per cent late, this element being also delayed with respect to the start signal by the maximum amount in each channel in the connexion;

(b) the receiving teleprinter were 1 per cent fast;

(c) the pick-up time of the receiving clutch happened to be a minimum; and

(d) all the small irregularities in the mechanical motions acted cumulatively to bring the selecting instant forward by the maximum amount possible.

In practice it is found that the simultaneous occurrence of all extreme conditions is sufficiently rare to be neglected.

Another point to remember is that when we say, for example, that the measured distortion for mixed signals on a channel is 10 per cent, we mean that over the period of observation the maximum value noted for the relative displacement of any pair of signals is 10 per cent; and not necessarily that all signals are distorted by 10 per cent. On the other hand if the margin of a teleprinter is given as 40 per cent, it means that when all signals are consistently distorted by 40 per cent the teleprinter will still continue to give consistently accurate reception.

Distortion Limits for Teleprinter Networks and Switching Services. It is generally accepted that where the distortion of the individual links is relatively small, the most probable value of the distortion on a multi-link connexion is equal to the square root of the sum of the squares of the distortion on each link in the connexion. Using this assumption as a basis, it is possible to design a switching plan for a network having various types of links by giving each link a transmission coefficient proportional to the square of the maximum distortion likely to be experienced on the link under normal conditions. It is then arranged that the sum of the coefficients for a through connexion shall not exceed the coefficient corresponding to the square of the maximum permissible overall distortion.

This method is useful where the network is made up of links of various types, such as voice-frequency channels on underground cables, high-frequency carrier on overhead lines, sub-audio telegraph circuits, etc.

An alternative to the foregoing method is to arrange for all

links in the system to have a similar transmission performance, when it is only necessary to fix the maximum number of links which may be connected in tandem, irrespective of the length and type of the individual links. This gives a high degree of flexibility, and is the method adopted in Great Britain where all long-distance circuits are voice-frequency channels in underground cables, and physical circuits are usually short.

Typical adjustment and design limits used in this way are as follows—

TELEPRINTERS
(Maximum values after adjustment)

Speed error	0·5 per cent
Transmitter distortion . .	5 ,, ,,

VOICE-FREQUENCY CHANNELS
(Maximum distortion after adjustment)

1 channel	10 per cent
2 channels in tandem . .	14 ,, ,,
3 channels ,, ,, . . .	17 ,, ,.
4 channels ,. ,, . . .	20 ,, ,,

PHYSICAL CIRCUITS
(Maximum distortion)

Wholly physical point-to-point circuits	25 per cent
Physical links in mixed V.F. and physical connexions . .	10 ,, ,,

On the face of it, if the teleprinter margin is, say, 35 per cent, the limiting figures almost balance on a four-link switched connexion as follows :

Transmitter		Margin	35 per cent
distortion	5 per cent	Reduction for 1	
Line distortion	20 ,, ,,	per cent speed difference	5 ,, ,,
Total	25 ,. ,,	Total	30 ,, ,,

However, for the reasons already explained the margin and distortion figures are not, strictly speaking, directly comparable, and in practice the law of probability introduces a safety factor which covers the normal falling away from test limits which occurs between the routine readjustments of the equipment.

The design limits for distortion in respect of physical circuits can be translated into the maximum permissible lengths for the various classes of circuit. By suitable distortion tests on typical circuits, limiting lengths can be established for single-wire circuits without receiving relays, single-wire circuits using receiving relays, and loop circuits using receiving relays. These limits are then used for the purpose of routing circuits and a completely standardized network results. Since physical circuits for inclusion in multi-link connexions have the same limiting distortion as a V.F. channel they may be regarded as equivalent.

The effect of short local physical extensions from voice-frequency systems is neglible and disregarded.

Regenerative Telegraph Repeaters are repeaters which accept distorted telegraph signals in a similar manner to an ordinary receiving telegraph instrument, but instead of converting the signals into printed characters or a perforated tape, retransmit them as undistorted signals. Using such repeaters the number of linked sections can be increased without limit. For example, an eight-link connexion could be operated, provided a regenerative repeater were included between the fourth and fifth sections. Regenerative repeaters can also be used on circuits which exceed the limits laid down for single sections, and such circuits can then be included in the switching network.

EXPLANATORY NOTE CONCERNING MATHEMATICAL NOTATIONS

The method which has been used for setting up some of the mathematical expressions in this book is one which economizes space, cheapens production, and incidentally improves appearance by avoiding irregular spacings between lines. To this end the solidus (/) is used in place of the horizontal rule for denoting fractions, and brackets instead of the horizontal bar, in conjunction with the root sign, thus:

$$a/b \text{ instead of } \frac{a}{b} \text{ and}$$

$$\sqrt{(a+b)} \text{ instead of } \sqrt{a+b}.$$

The following examples further illustrate the method:

$$(1 - \omega^2 LC)^2/n^2\omega^2 LC = \frac{(1 - \omega^2 LC)^2}{n^2\omega^2 LC}$$

$$(Vs/Z_0) \sinh \gamma x = \frac{Vs}{Z_0} \sinh \gamma x.$$

$$R/2 + y = \frac{R}{2} + y$$

$$R/(2 + y) = \frac{R}{2 + y}$$

$$(E/R)e^{(4/RC)t} = \frac{E}{R} e^{-\frac{4}{RC}t}$$

$$\sqrt{(RG)} \cdot l = \sqrt{RG} \cdot l$$

$$\sqrt{[(R + j\omega L)/(G + j\omega C)]} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$[(5 + \sqrt{5})/2](t/C_n R_n) = \frac{5 + \sqrt{5}}{2} \cdot \frac{t}{C_n R_n}$$

The method calls perhaps, at first, for some added care in reading the formulae, but once the reader is aware of the method in use no difficulty is likely to be experienced.

APPENDIX

PROOFS OF THE FORMULAE GIVEN IN THE MAIN TEXT

Charge and Discharge of a Condenser. Referring to Fig. 45 an e.m.f. of E volts is applied to a condenser C through a non-inductive resistance R.

Taking note of the fact that the voltage across the condenser
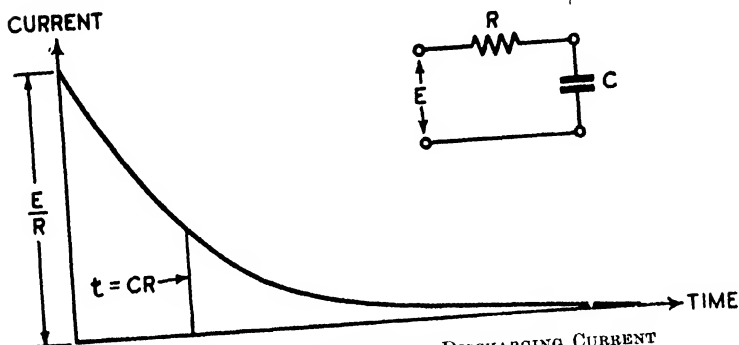
FIG. 45. CURVE OF CHARGING OR DISCHARGING CURRENT FOR CONDENSER AND RESISTANCE

is at all times equal to Q/C or $(1/C) \int i dt$, we can write—

$$Ri + (1/C) \int i dt = E$$

where i = value of the current at any instant; and
t = time in seconds from instant of switching on the e.m.f.

Differentiating gives

$$R(di/dt) + (1/C)i = 0,$$

a differential equation, the solution of which is

$$i = Ae^{-(t/RC)}$$

where A is some constant determined from the known condition at time $t = 0$.

When $t = 0$, $i = E/R$ because there is no charge on the condenser.

$\therefore A = E/R$ and

$$i = (E/R)e^{-(t/RC)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The significance of this formula has already been explained and the curve for the current is given in Fig. 45.

For the discharge of the condenser through the resistance R,
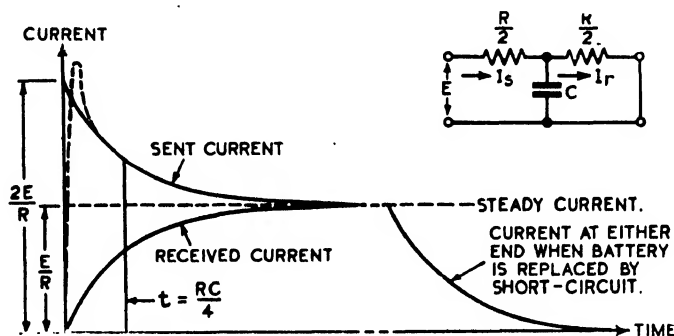
FIG. 46. CURVES OF SENT AND RECEIVED CURRENT
FOR SHORT ARTIFICIAL LINE

(The dotted curve shows the effect of the distributed capacitance near the sending end and the small amount of inductance which would be present in a real line.)

because the voltage across the resistance is at all times equal to that across the condenser, we have—

$$Ri + (1/C)\int i\,dt = 0$$

the solution of which is the same as before, namely—

$$i = (E/R)e^{-(t/RC)}.$$

Single-section Artificial Line. Referring to Fig. 46, the T network shown is a single-section artificial line representing a real line of negligible inductance and leakance having a resistance of R ohms and a total capacitance to earth of C farads.

Suppose an e.m.f. of E volts and negligible internal resistance is applied to one end, the other end being short-circuited. It is desired to find the current at each end of the line.

If V is the voltage across the condenser at any instant we have—

$$\frac{E-V}{R/2} = I_s = C\frac{dV}{dt} + I_r$$

where $C(dV/dt)$ is the current flowing into the condenser at any instant.

$$\therefore \qquad \frac{E-V}{R/2} = C\frac{dV}{dt} + \frac{2V}{R}$$

and
$$E - V_{\text{.}} = \frac{CR}{2}\frac{dV}{dt} + V$$

or
$$\frac{E}{2} = V + \frac{CR}{4}\frac{dV}{dt}$$

the solution of which is

$$V = (E/2) + Ae^{-(4/CR)t}$$
$$V = 0 \text{ when } t = 0 ;$$
$$\therefore A = -(E/2) \text{ and}$$
$$V = (E/2)(1 - e^{-(4/CR)t})$$
$$\therefore I_s = (E/R)(1 + e^{-(4/CR)t}) \qquad . \qquad . \qquad . \qquad . \qquad . \quad (2)$$
$$\text{and } I_r = (E/R)(1 - e^{-(4/CR)t}) \qquad . \qquad . \qquad . \qquad . \qquad . \quad (3)$$

(*Note.* When t is very great $V = E/2$ and $I_s = I_r = E/R$.)

If when the condenser is charged the battery is removed and the line short-circuited, it will discharge through the two resistances in parallel and from equation (1) we have

$$I_s + I_r = (E/2) \cdot (4/R) e^{-(4/CR)t}$$
$$\text{and } I_s = I_r = (E/R)e^{-(4/CR)t} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (4)$$

Curves drawn from equations (2), (3), and (4) are given in Fig. 46.

Single-section Artificial Line with Terminal Resistances. If in the preceding case the voltage is applied to the line through a resistance of x ohms, and the receiving end of the line is terminated on a resistance of y ohms, the solution for the currents is as follows—

$$\frac{E-V}{(R/2)+x} = C\frac{dV}{dt} + \frac{V}{(R/2)+y}$$

$$\therefore (E-V)[(R/2)+y)] = [(R/2)+x][(R/2)+y]$$
$$C(dV/dt) + V[(R/2)+x]$$

$$\therefore E\frac{(R/2)+y}{R+x+y} = \frac{[(R/2)+x][(R/2)+y]}{R+x+y}C\frac{dV}{dt} + V.$$

$$\therefore \qquad V = \frac{(R/2)+y}{R+x+y}E + Ae^{-\frac{R+x+y}{[(R/2)+x][(R/2)+y]}\cdot\frac{t}{C}}$$

$V = 0$ when $t = 0$ so that

$$V = \frac{(R/2) + y}{R + x + y} E \left[1 - e^{- \frac{R + x + y}{[(R/2) + x][(R/2) + y]} \cdot \frac{t}{C}} \right]$$

whence

$$I_r = \frac{E}{R + x + y} \left[1 - e^{- \frac{R + x + y}{[(R/2) + x][(R/2) + y]} \cdot \frac{t}{C}} \right]$$

or

$$I_r = \frac{E}{R + x + y} \left[1 - e^{- \frac{R + x + y}{R + 2(x + y) + (4xy/R)} \cdot \frac{4}{CR} \cdot t} \right] \quad (5)$$

Growth and Decay of Current in an Inductive Circuit. If an e.m.f. of E volts is applied to a circuit having a resistance of R ohms and an inductance of L henrys, the sum of the p.d. across the resistance and the opposing e.m.f. generated in the inductance is at all times equal to the applied e.m.f. Since the opposing e.m.f. is, at any instant, proportional to the rate of change of the current, we have

$$E = Ri + L \, (di/dt)$$

where $i =$ the current at any instant.

Whence
$$i + (L/R) \, (di/dt) = E/R.$$
and
$$i = E/R + Ae^{-(R/L)t}.$$

For $t = 0$, $i = 0$, so that $A = -(E/R)$ and
$$i = (E/R) (1 - e^{-(R/L)t}) \quad . \quad . \quad . \quad (6)$$

If the battery is replaced by a short circuit after a relatively long interval of time, we have

$$Ri + L(di/dt) = 0,$$
and
$$i = Ae^{-(R/L)t}.$$

When $t = 0$, that is, at the instant of short-circuiting, we have

$i = E/R$ and therefore $A = E/R$,

and $i = (E/R)e^{-(R/L)t}$ . . . . (7)

which is the equation for the decay of the current.

Curves drawn from equations (6) and (7) are given in Fig. 47.
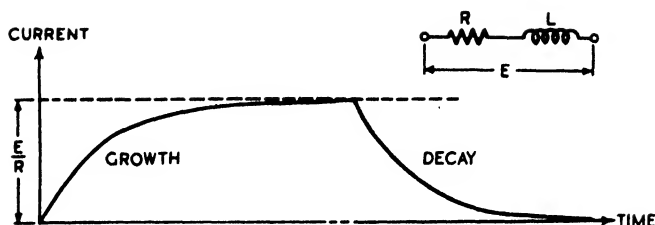
FIG. 47. CURVES OF CURRENT IN AN INDUCTIVE CIRCUIT

Closed Circuit Working. The solution for the circuit and artificial line shown in Fig. 48 is as follows—
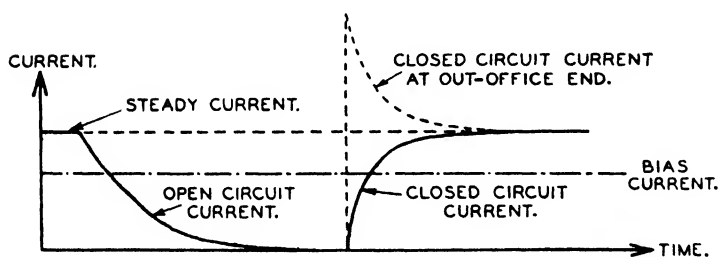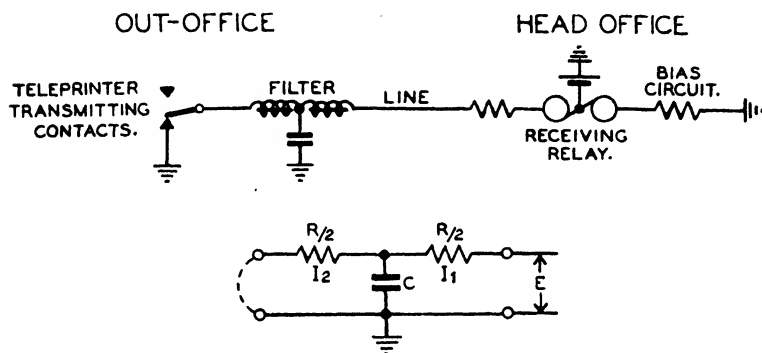
FIG. 48. CLOSED CIRCUIT WORKING ON SEND LEG OF TELEPRINTER
PRIVATE WIRE
Local end

When the circuit is closed the current flowing is given by $I_1 = E/R$, and the voltage across condenser C is therefore equal to $E/2$.

When the far end of the circuit is opened, the condenser charges up to the full voltage of the battery.

We have

$$E = (R/2)I_1 + (1/C)\int I_1 dt$$

and differentiating

$$(R/2)(dI_1/dt) + (1/C)I_1 = 0$$

So that

$$I_1 = Ae^{-(2/CR)t}.$$

When $t = 0$, that is, at the instant of opening the line, $I_1 = E/R$.

$$\therefore A = E/R$$

and $I_1 = (E/R)e^{-(2/CR)t}$ . . . . . . (8)

If the line is now short-circuited, the condenser discharges to the voltage $E/2$ again.

If V is the voltage across the condenser at any instant,

$$\frac{E-V}{R/2} = I_1 = C\frac{dV}{dt} + I_2$$

$$\therefore \qquad \frac{E-V}{R/2} = C\frac{dV}{dt} + \frac{2V}{R}$$

$$\therefore \qquad E/2 = (CR/4)(dV/dt) + V.$$

This is the same differential equation as was obtained for the normal single-section artificial line, and the solution is

$$V = E/2 + Ae^{-(4/CR)t}.$$

The known conditions are however different in this case, because for $t = 0$, that is, the instant of short-circuiting the line, the condenser is charged to the full voltage of the battery and V is equal to E.

$$\therefore A = E/2$$

and $V = (E/2)(1 + e^{-(4/CR)t})$.

$$\therefore I_1 = \frac{E-V}{R/2} = \frac{E}{R}(1 - e^{-(4/CR)t})$$ . . . . (9)

and $I_2 = \dfrac{E}{R}(1 + e^{-(4/CR)t})$ . . . . . (10)

The curves obtained from equations (8), (9), and (10) are given in Fig. 48.

Steady-state D.C. Solution for a Leaky Line. Fig. 49 repre-
sents a single-wire line, having uniformly distributed resistance
and leakance. The capacitance has been neglected as the
steady current condition only is being considered.

The problem is to obtain an expression for the current when
a steady voltage is applied to the line and sufficient time has
elapsed for the current to assume a steady value. That is
to say, all transient effects are neglected.
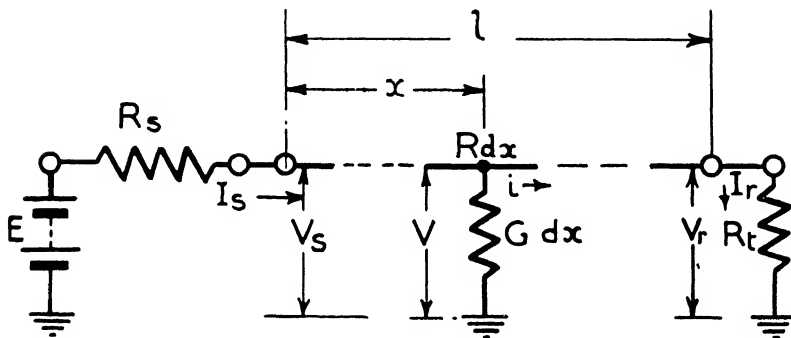
FIG. 49. LINE WITH UNIFORMLY DISTRIBUTED LEAKANCE

Let R = resistance per unit length (ohms);

G = leakance per unit length (mhos);

R_s = external resistance at sending end;

R_t = terminating resistance at receiving end;

v = voltage at distance x from sending end;

i = current at distance x from sending end.

If Rdx and Gdx represent the resistance and leakance of
an elementary length of line the decrement of line current
for such a length at any point distance x from the sending end
is given by $vGdx$, and the decrement of voltage, or voltage
drop, by $iRdx$, whence

$$\left. \begin{array}{l} - (di/dx) = vG \\ - (dv/dx) = iR \end{array} \right\} \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \ (11)$$

and differentiating

$$- (d^2i/dx^2) = (dv/dx)\, G$$
$$- (d^2v/dx^2) = (di/dx)\, R.$$

Substituting we get

$$d^2i/dx^2 = iRG.$$
$$d^2v/dx^2 = vRG.$$

The solution of differential equations of this type is of the form—

$$\left.\begin{aligned} i &= A_1 e^{-\sqrt{(RG)}.x} + A_2 e^{\sqrt{(RG)}.x} \\ v &= A_3 e^{-\sqrt{(RG)}.x} + A_4 e^{\sqrt{(RG)}.x} \end{aligned}\right\} \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \quad (12)$$

which can be easily confirmed by differentiating.

The constants A_1, A_2, A_3, A_4 are found from the known terminal conditions as follows—

When $x = 0$,

$$i = I_s \text{ and } v = V_s.$$

$$\therefore \quad \left.\begin{aligned} I_s &= A_1 + A_2 \\ \text{and} \quad V_s &= A_3 + A_4 \end{aligned}\right\} \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \quad (13)$$

Differentiating equations (12), substituting in equation (11) and again putting $x = 0$, we have

$$V_s G = \sqrt{(RG)} . A_1 - \sqrt{(RG)} . A_2,$$
$$I_s R = \sqrt{(RG)} . A_3 - \sqrt{(RG)} . A_4.$$

Writing $R_o = \sqrt{(R/G)}$,

$$V_s/R_o = A_1 - A_2,$$
$$\text{and } I_s R_o = A_3 - A_4,$$

whence from equation (13) we obtain

$$A_1 = \tfrac{1}{2}[I_s + (V_s/R_o)], \; A_2 = \tfrac{1}{2}[I_s - (V_s/R_o)],$$
$$A_3 = \tfrac{1}{2}(V_s + I_s R_o), \; A_4 = \tfrac{1}{2}(V_s - I_s R_o)$$
$$\therefore \quad i = [(I_s/2) \, (e^{-\sqrt{(RG)}.x} + e^{\sqrt{(RG)}.x})]$$
$$+ [(V_s/2R_o) \, (e^{-\sqrt{(RG)}.x} - e^{\sqrt{(RG)}.x})]$$
$$\text{and} \quad v = [(V_s/2) \, (e^{-\sqrt{(RG)}.x} + e^{\sqrt{(RG)}.x})]$$
$$+ [(I_s R_o/2) \, (e^{-\sqrt{(RG)}.x} - e^{\sqrt{(RG)}.x}).]$$

t is convenient here to introduce the hyperbolic functions*
iimplify notation, as follows

$$i = I_s \cosh\left[\sqrt{(RG)} \cdot x\right] - (V_s/R_o) \sinh\left[\sqrt{(RG)} \cdot x\right] \left.\right\}$$
$$v = V_s \cosh\left[\sqrt{(RG)} \cdot x\right] - I_s R_o \sinh\left[\sqrt{(RG)} \cdot x\right] \left.\right\} \quad .(14)$$

These are the fundamental transmission equations from
ich the current and voltage at any point in the circuit can
determined. They should be compared with those for an
pressed sinusoidal e.m.f. which are given later.
By putting $x = l$ in equations (14), the received current and
tage are given by—

$$I_r = I_s \cosh\left[\sqrt{(RG)} \cdot l\right] - (V_s/R_o) \sinh\left[\sqrt{(RG)} \cdot l\right]$$
$$V_r = V_s \cosh\left[\sqrt{(RG)} \cdot l\right] - I_s R_o \sinh\left[\sqrt{(RG)} \cdot l.\right]$$

Noting that $I_r = V_r/R_t$ and eliminating I_s gives

$$I_r = \frac{V_s}{R_o \sinh\left[\sqrt{(RG)} \cdot l\right] + R_t \cosh\left[\sqrt{(RG)} \cdot l\right]} \qquad . (15)$$

Eliminating I_r gives

$$I_s = \frac{V_s}{R_o} \cdot \frac{R_o \cosh\left[\sqrt{(RG)} \cdot l\right] + R_t \sinh\left[\sqrt{(RG)} \cdot l\right]}{R_t \cosh\left[\sqrt{(RG)} \cdot l\right] + R_o \sinh\left[\sqrt{(RG)} \cdot l\right]} \qquad . (16)$$

With no external resistance at the sending end $V_s = E$ and

$$I_r = \frac{E}{R_o \sinh\left[\sqrt{(RG)} \cdot l\right] + R_t \cosh\left[\sqrt{(RG)} \cdot l\right]} \qquad . (17)$$

When R_s is not equal to zero we have $V_s = E - I_s R_s$, and
bstituting in equation (17) gives

$$I_r = \frac{E - I_s R_s}{R_o \sinh\left[\sqrt{(RG)} \cdot l\right] + R_t \cosh\left[\sqrt{(RG)} \cdot l\right]}$$

Making use of equation (16) we get

$$= \frac{E[R_o \sinh\left[\sqrt{(RG)} \cdot l\right] + R_t \cdot \cosh\left[\sqrt{(RG)} \cdot l\right]]}{(R_t + R_s)\cosh\left[\sqrt{(RG)} \cdot l\right] + [R_o + (R_s R_t/R_o)]\sinh\left[\sqrt{(RG)} \cdot l\right]}$$

* Certain hyperbolic functions are analogous to the sine, cosine, tangent,
:., functions of trigonometry. They are called *hyperbolic sine, hyperbolic*
ine, etc., and are written *sinh, cosh*, and so on. They represent a convenient
ithod of handling certain exponential functions. Graphs of sinh θ, cosh θ,
d tanh θ are given in Fig. 18 of Section 3.

$$\text{Sinh } \theta = (e^\theta - e^{-\theta})/2.$$
$$\text{Cosh } \theta = (e^\theta + e^{-\theta})/2.$$
$$\text{Tanh } \theta = \text{Sinh } \theta / \text{Cosh } \theta = (e^\theta - e^{-\theta})/(e^\theta + e^{-\theta})$$
$$\text{Cosh}^2 \theta - \text{Sinh}^2 \theta = 1$$

and

$$I_r = \cfrac{E}{(R_t + R_s)\cosh[\sqrt{(RG)}.l] + [R_o + (R_s R_t/R_o)]\sinh[\sqrt{(RG)}.l]}$$

Transient Solution for a Leaky Line. To obtain a general solution for the transient conditions as has been done for the steady-state condition is beyond the scope of the present treatment. A solution suitable to the case treated in Section
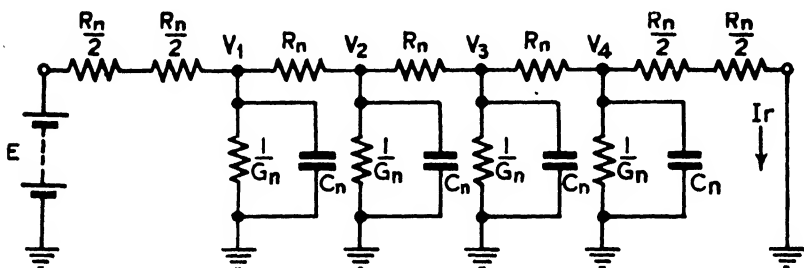
FIG. 50. ARTIFICIAL LINE WITH LEAKANCE AND CAPACITANCE

2 can, however, be fairly readily obtained by reference to the artificial line represented in Fig. 50.

R_n = conductor resistance of each section (ohms);

C_n = capacitance of each section (farads);

G_n = leakance of each section (mhos);

V_1, V_2, etc. = voltage at each junction;

I_r = received current;

$R_n/2$ = value of terminal resistances.

Noting that the current passing from the battery is at all times equal to the sum of the received current and the current passing to earth at each leakage path we can write—

$$(E - V_1)/R_n = G_n[V_1 + V_2 + V_3 + V_4] + C_n(d/dt)[V_1 + V_2 + V_3 + V_4] + (V_4/R_n).$$

$$\therefore E = R_n[G_n + C_n(d/dt)](V_1 + V_2 + V_3 + V_4) + V_1 + V_4.$$

Now $(V_3 - V_4)/R_n = C_n V_4 + C_n(dV_4/dt) + (V_4/R_n).$

$$\therefore \qquad V_3 = R_n G_n V_4 + R_n C_n(dV_4/dt) + 2V_4$$

and $(dV_3/dt) = R_n G_n(dV_4/dt) + R_n C_n(d^2V_4/dt^2) + 2(dV_4/dt).$

Similarly

$$(V_2 - V_3)/R_n = G_n V_3 + C_n(dV_3/dt) + (V_3 - V_4)R_n.$$
$$(V_1 - V_2)/R_n = G_n V_2 + C_n(dV_2/dt) + (V_2 - V_3)/R_n.$$

Manipulation of the foregoing equations yields a differential equation of the fourth order in V_4, the solution of which is

$$V_4 = \frac{E}{a \cdot b \cdot c \cdot d} + A_1 e^{-(a/C_n R_n)t} + A_2 e^{-(b/C_n R_n)t}$$
$$+ A_3 e^{-(c/C_n R_n)t} + A_4 e^{-(d/C_n R_n)t}$$

where
$$a = R_n G_n + \tfrac{1}{2}(3 - \sqrt{5});$$
$$b = R_n G_n + \tfrac{1}{2}(5 - \sqrt{5});$$
$$c = R_n G_n + \tfrac{1}{2}(3 + \sqrt{5});$$
$$d = R_n G_n + \tfrac{1}{2}(5 + \sqrt{5}).$$
$$A_1 = -\ 1/a(5 + \sqrt{5});$$
$$A_2 = \ \ \ \ 1/b(5 - \sqrt{5});$$
$$A_3 = -\ 1/c(5 - \sqrt{5});$$
$$A_4 = \ \ \ \ 1/d(5 + \sqrt{5}).$$

I_r is obtained by dividing the foregoing equation by R_n, giving—

$$I_r = \frac{E}{R_n abcd}\left\{ 1 - \frac{bcd}{5 + \sqrt{5}} e^{-(a/C_n R_n)t} + \frac{acd}{5 - \sqrt{5}} e^{-(b/C_n R_n)t} \right.$$
$$\left. - \frac{abd}{5 - \sqrt{5}} e^{-(c/C_n R_n)t} + \frac{abc}{5 + \sqrt{5}} e^{-(d/C_n R_n)t} \right\}$$

The solution for the case where the leakance is negligible is obtained by putting $G_n = 0$ in the foregoing equation, giving—

$$I_r = \frac{E}{5R_n}\left\{ 1 - \frac{5}{5 - \sqrt{5}} e^{-[(3 - \sqrt{5})/2] \cdot t/(C_n R_n)} + \frac{1}{3 - \sqrt{5}} e^{-[(5 - \sqrt{5})/2 \cdot (t/C_n R_n)} \right.$$
$$\left. - \frac{5}{5 + \sqrt{5}} e^{-[(3 + \sqrt{5})/2] \cdot (t/C_n R_n)} + \frac{1}{3 + \sqrt{5}} e^{-[(5 + \sqrt{5})/2] \cdot (t/C_n R_n)} \right\}$$

Line with Localized Insulation Resistance Fault. This case is illustrated in Fig. 51. The fault resistance is assumed to be low compared to the distributed insulation resistance.

Referring to Fig. 51—

R = resistance to the fault including any terminal resistance;

R_1 = resistance beyond the fault including the resistance at the receiving end;

F = fault resistance;

I_r = received current.
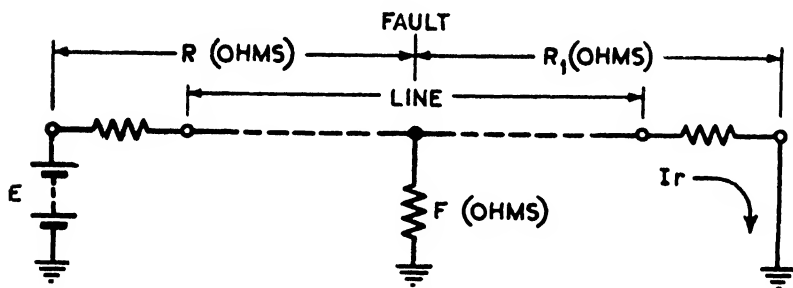
FIG. 51. LINE WITH LOCALIZED INSULATION RESISTANCE FAULT

The voltage at the fault is equal to $I_r R_1$ and the fault current is therefore given by $I_r R_1/F$.

Thus we have—

$$(E - I_r R_1)/R = (I_r R_1/F) + I_r.$$

$$\therefore \quad I_r[1 + (R_1/F) + (R_1/R)] = E/R$$

or $\quad I_r(FR + R_1 R + R_1 F) = EF,$

and $\quad I_r = EF/[RR_1 + F(R + R_1)].$

Derivation of the Classical A.C. Transmission Equations. The method of deriving these equations is essentially similar to that used for obtaining the steady-state solution in the somewhat simpler case of the leaky line which was given earlier.

Fig. 52 refers to transmission circuits with uniformly distributed resistance, inductance, capacitance, and leakance. The primary constants are R, L, C, and G per unit length. In the case of the loop circuit, the resistance and inductance are equally divided between the two wires. The following equations apply equally to each circuit.

It is assumed that a sinusoidal e.m.f., $E \sin \omega t$, of frequency f, where $\omega = 2\pi f$, is applied to one end of the circuit, the other end being closed by an impedance Z_t.

The problem is to determine the voltage and current at any point in the circuit when the steady-state conditions have been reached, and all transient effects due to switching on the e.m.f. have subsided.

Consider an elementary length of line δx, distant x from the

SINGLE – WIRE CIRCUIT

LOOP CIRCUIT
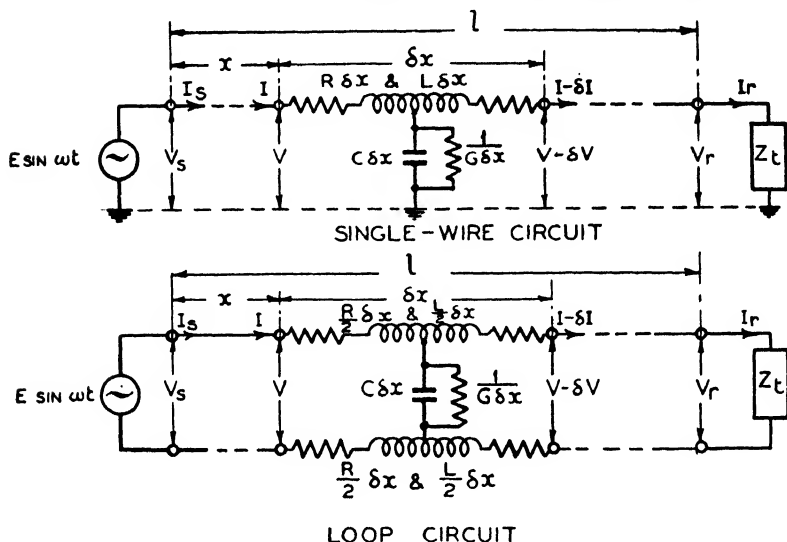
FIG. 52. TRANSMISSION CIRCUITS

sending end. The constants of this length of line are $R\delta x$, $L\delta x$, $G\delta x$, and $C\delta x$.

The change in the line current due to the current flowing through the capacitance and leakance of this elementary section of line is given by

$$- \delta I = VG\delta x + (dV/dt)\, C\delta x$$

the current passing into the capacitance being proportional to the rate of change of voltage.

Similarly the change of voltage is given by the voltage drop across the resistance and the opposing e.m.f. induced in the inductance, this e.m.f. being proportional to the rate of change of current.

$$- \delta V = IR\delta x + (dI/dt)\, L\delta x.$$

In the limit when δx is infinitesimally small we have

$$\left. \begin{array}{l} - (dI/dx) = VG + (dV/dt)\, C \\ - (dV/dx) = IR + (dI/dt)\, L \end{array} \right\} \quad . \quad \quad . \quad \quad . \quad (18)$$

The negative signs before the differentials indicate that the current and voltage decrease as x increases.

Now V and I are sinusoidal; therefore

$$dV/dt = j\omega V, \text{ and } dI/dt = j\omega I.$$

Thus

$$\left. \begin{array}{l} -(dI/dx) = V(G + j\omega C) \\ -(dV/dx) = I(R + j\omega L) \end{array} \right\} \qquad . \qquad . \qquad . \qquad . \qquad (19)$$

Equations (18) are the fundamental circuit equations, while the equations (19) are the special case of these fundamental equations for a sinusoidal wave form.

Differentiating gives

$$-(d^2I/dx^2) = (dV/dx)(G + j\omega C);$$
$$-(d^2V/dx^2) = (dI/dx)(R + j\omega L)$$

Substituting the values for dV/dx and dI/dx gives

$$d^2I/dx^2 = I(R + j\omega L)(G + j\omega C);$$
$$d^2V/dx^2 = V(R + j\omega L)(G + j\omega C);$$
$$\text{or } \left. \begin{array}{l} d^2I/dx^2 = p^2I \\ d^2V/dx^2 = p^2V \end{array} \right\} \qquad . \qquad . \qquad . \qquad . \qquad (20)$$

where $p = \sqrt{[(R + j\omega L)(G + j\omega C)]}$ and is called the *propagation constant*.

The solution of differential equations of the type of equations (20) is of the form—

$$\left. \begin{array}{l} I = A_1 e^{-px} + A_2 e^{px} \\ V = A_3 e^{-px} + A_4 e^{px} \end{array} \right\} \qquad . \qquad . \qquad . \qquad . \qquad (21)$$

which can be readily proved by differentiating.

The constants A_1, A_2, A_3, A_4 are found from the known terminal conditions as follows—

When $x = 0$

$$I = I_s \text{ and } V = V_s.$$
$$\left. \begin{array}{l} \therefore \quad I_s = A_1 + A_2 \\ \text{and} \quad V_s = A_3 + A_4 \end{array} \right\} \qquad . \qquad . \qquad . \qquad . \qquad (22)$$

Differentiating equations (21) gives

$$dI/dx = -pA_1 e^{-px} + pA_2 e^{px};$$
$$dV/dx = -pA_3 e^{-px} + pA_4 e^{px}.$$

Putting $x = 0$ and using equations (19) gives

$$V_s(G + j\omega C) = pA_1 - pA_2;$$
$$I_s(R + j\omega L) = pA_3 - pA_4;$$

and
$$\left. \begin{array}{l} V_s/Z_o = A_1 - A_2 \\ I_s Z_o = A_3 - A_4 \end{array} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $Z_o = \sqrt{[(R + j\omega L)/(G + j\omega C)]}$ and is called the *characteristic impedance.*

Equations (22) and (23) give

$$A_1 = \tfrac{1}{2}[I_s + (V_s/Z_o)], A_2 = \tfrac{1}{2}[I_s - (V_s/Z_o)]$$
$$A_3 = \tfrac{1}{2}(V_s + I_s Z_o), A_4 = \tfrac{1}{2}(V_s - I_s Z_o).$$

We have therefore

$$I = \tfrac{1}{2} I_s(e^{-px} + e^{px}) + \tfrac{1}{2} (V_s/Z_o) (e^{-px} - e^{px});$$
$$V = \tfrac{1}{2} V_s(e^{-px} + e^{px}) + \tfrac{1}{2} I_s Z_o(e^{-px} - e^{px});$$

and finally

$$\left. \begin{array}{l} I = I_s \cosh px - (V_s/Z_o) \sinh px \\ V = V_s \cosh px - I_s Z_o \sinh px \end{array} \right\} \quad . \quad . \quad . \quad (24)$$

These are the fundamental a.c. transmission equations giving the steady-state values of the current and voltage at any point in the circuit. It should be remembered that the equations only apply to the case where the current and voltage are sinusoidal.

The application of the equations is given in the main text.

Recurrent Network Theory. *To deduce expressions for the characteristic impedance and propagation constant.*

Fig. 53 represents any three sections of a recurrent ladder network having an infinitely large number of sections.

It is apparent that as there is an infinite number of sections, if the network is cut at a given point between any two sections the impedance looking towards the distant end will always be the same. If the networks are cut midway between two shunt sections the mid-series characteristic impedance will be measured. This impedance is indicated by Z_{o1} in the following.

Let i_{n-1} = current entering $(n - 1)$th section;

i_n = current entering nth section;

i_{n+1} = current entering $(n + 1)$th section;

We have

$i_{n-1} - i_n =$ current flowing in shunt impedance of $(n-1)$th section.

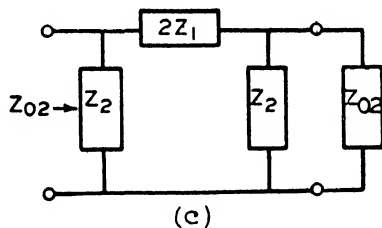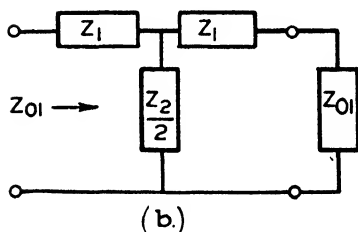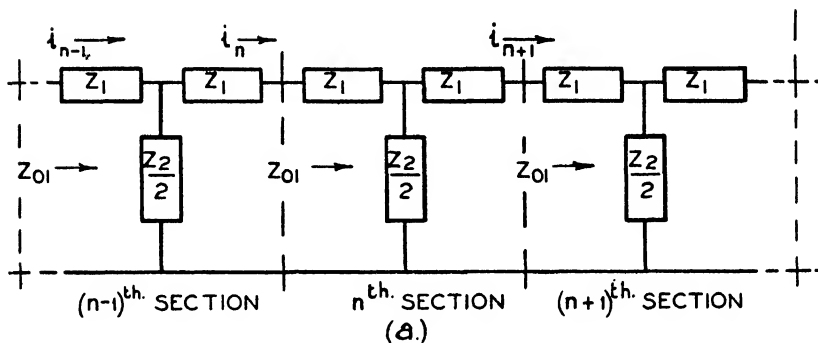$i_n - i_{n+1} =$ current flowing in shunt impedance of nth section.

FIG. 53. RECURRENT LADDER NETWORK

Thus

$$(i_{n-1} - i_n)(Z_2/2) = 2Z_1 i_n + (i_n - i_{n+1})(Z_2/2).$$

$$i_{n-1} - i_n = 4(Z_1/Z_2)i_n + i_n - i_{n+1}.$$

$$\therefore \quad \frac{i_{n-1}}{i_n} = 4\frac{Z_1}{Z_2} + 2 - \frac{i_{n+1}}{i_n}$$

Now n is only some arbitrary number, so that if we write $(n+1)$ for n, which is therefore quite legitimate, we have

$$i_{n-1}/i_n = i_{(n+1)-1}/i_{n+1} = i_n/i_{n+1}.$$

So that we can write

$$(i_{n-1}/i_n) + (i_n/i_{n-1}) = 4(Z_1/Z_2) + 2.$$

If P is the propagation constant per section we have

$$e^P = i_{n-1}/i_n.$$

$$\therefore \quad e^P + e^{-P} = 4(Z_1/Z_2) + 2.$$

Noting that $(e^P + e^{-P})/2 = \cosh P$ we have

$$\cosh P = 2(Z_1/Z_2) + 1 \quad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (25)$$

Also $i_1/i_2 = i_2/i_3 = i_3/i_4 = \ldots = e^P$,

where i_1, i_2, i_3, etc., are the currents entering the 1st, 2nd, 3rd, etc., sections.

Also $\quad i_1/i_{n+1} = e^{nP}$,

where i_{n+1} is the current entering the $(n+1)$th section, or alternatively the current received at the end of a network of n sections terminated in its characteristic impedance.

The mid series-characteristic impedance can be obtained by assuming a mid-series section terminated by its proper impedance as shown in Fig. 53 (b).

Thus

$$Z_{01} = Z_1 + \frac{\frac{1}{2}Z_2(Z_1 + Z_{01})}{\frac{1}{2}Z_2 + Z_1 + Z_{01}}$$

Whence

$$Z_{01} = \sqrt{(Z_1^2 + Z_1 Z_2)} \quad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (26)$$

Similarly the mid-shunt characteristic impedance can be found by assuming a mid-shunt section terminated as shown in Fig. 53 (c).

$$Z_{02} = \frac{Z_2\{2Z_1 + Z_2 Z_{02}/(Z_2 + Z_{02})\}}{Z_2 + 2Z_1 + Z_2 Z_{02}/(Z_2 + Z_{02})}$$

Whence

$$Z_{02} = Z_1 Z_2 / \sqrt{(Z_1^2 + Z_1 Z_2)} \quad . \qquad . \qquad . \qquad . \qquad . \quad (27)$$

When the e.m.f. applied to the network is sinusoidal the propagation constant is complex and consists, as in ordinary transmission theory, of an attenuation constant (A) and phase constant (B)

Thus $P = A + jB$

and $I_1/I_2 = e^A \times e^{jB}$.

or $I_1 = I_2 e^A \underline{/B}$

and $I_1 = I_n e^{nA} \underline{/nB}$.

The real portion A of the propagation constant gives the ratio of the magnitudes of the sent and received currents and

the phase constant B gives the phase difference between the sent and received currents.

Now $\cosh P = \cosh(A + jB)$

$$= \cosh A \cosh jB + \sinh A \sinh jB.$$

$\therefore \cosh P = 1 + (2Z_1/Z_2) = \cosh A \cos B + j \sinh A \sin B \quad (28)$

Equation (28) can be used to investigate the behaviour of filters.

Extension of Theory to the Case of Electric Wave Filters. In an ideal filter the impedances Z_1 and Z_2 are wholly reactive and the expression $1 + 2Z_1/Z_2$ for the propagation constant is always real.

Equating real and imaginary parts of equation (28) gives

$$\cosh A \cos B = 1 + (2Z_1/Z_2);$$

$$\sinh B \sin A = 0.$$

We have therefore either

$$\sinh A = 0.$$
$$\text{or } \sin B = 0.$$

When $\sinh A = 0$, $A = 0$ and there is therefore no attenuation through the filter. This corresponds to the pass range of the filter.

When $A = 0$, $\cosh A = 1$ (see Fig. 18) and therefore

$$\cos B = 1 + (2Z_1/Z_2).$$

When $\sin B = 0$, $\cos B = 1$ and

$$\cosh A = 1 + (2Z_1/Z_2).$$

This corresponds to the attenuating range when the currents are attenuated, but there is no change of phase.

The conditions for each of the various types of filters can be found by substituting the proper values of Z_1 and Z_2.

Low-pass Filter.

$$Z_1 = j\omega L;$$
$$Z_2 = 1/j\omega C.$$
$$\cosh P = 1 + (2Z_1/Z_2) = 1 - 2\omega^2 LC.$$

In the pass range we have

$$\cos B = 1 - 2\omega^2 LC.$$

A limiting condition occurs when ω has reached such a value that $\omega^2 LC = 1$, when we have

$$\cos B = -1.$$

This is the condition for cut-off already given in the main text. Thus from $\omega = 0$ to $\omega = 1/\sqrt{(LC)}$, $\cos B$ varies from $+1$ to -1. For higher values of ω the expression $1 - 2\omega^2 LC$ exceeds -1, and the conditions are changed ($\cos B$ cannot exceed ± 1) and $\cos B$ reverts to $+1$ when $\sin B = 0$ and therefore

$$\cosh A = 1 - 2\omega^2 LC.$$

High-pass Filter.

$$Z_1 = 1/j\omega C\,;$$
$$Z_2 = j\omega L.$$
$$\cosh P = 1 + (2Z_1/Z_2) = 1 - (2/\omega^2 LC).$$

The conditions for cut-off are the same as before, except that it is clear that for values of ω less than $\omega = 1/\sqrt{(LC)}$ the expression $1 - (2/\omega^2 LC)$ is greater than -1 and therefore the attenuating range is from $\omega = 0$ up to $\omega = 1/\sqrt{(LC)}$.

Band-pass Filter.

$$Z_1 = \frac{j[\omega L - (1/\omega C)]}{n}$$

$$Z_2 = \frac{n}{j} \cdot \frac{L/C}{\omega L - 1/\omega C}$$

$$\cosh P = 1 + 2Z_1/Z_2 = 1 - [2(1 - \omega^2 LC)^2/n^2 \omega^2 LC].$$

The conditions for cut-off are

$$(1 - \omega^2 LC)^2/n^2 \omega^2 LC = 1,$$
$$\text{i.e. } 1 - 2\omega^2 LC + \omega^4 L^2 C^2 = n^2 \omega^2 LC,$$
$$\text{or} \quad \omega^4 L^2 C^2 - (n^2 + 2)\omega^2 LC + 1 = 0.$$
$$\therefore \quad \omega^2 LC = \frac{n^2 + 2 \pm \sqrt{[(n^2 + 2)^2 - 4]}}{2}$$

This gives two positive values for ω namely

$$\omega_1 = [\sqrt{(n^2 + 4)} - n]/2\sqrt{(LC)}\,;$$
$$\omega_2 = [\sqrt{(n^2 + 4)} + n]/2\sqrt{(LC)}\,;$$
$$\text{whence} \quad \omega_1 \omega_2 = 1/LC\,;$$
$$\omega_2 - \omega_1 = n/\sqrt{(LC)}\,;$$

or $\quad \sqrt{(f_1 f_2)} = 1/2\pi\sqrt{(LC)}$,

and $\quad (f_2 - f_1)/\sqrt{(f_2 f_1)} = n$.

It can readily be shown that when ω falls between the values ω_1 and ω_2 the expression for cosh P is less than 1 and therefore the attenuation is zero and cos B passes from -1 through $+1$ to -1 while B passes from $-180°$ through $0°$ to $+180°$.

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